



# Predicate Logic



# Questions yet to be answered:

- Aristotle or the Stoics?
- Aristotle or Boole?
- Is Logicism true?
- How do we know Stoic argument forms are actually valid?

###

goals for unit IV:

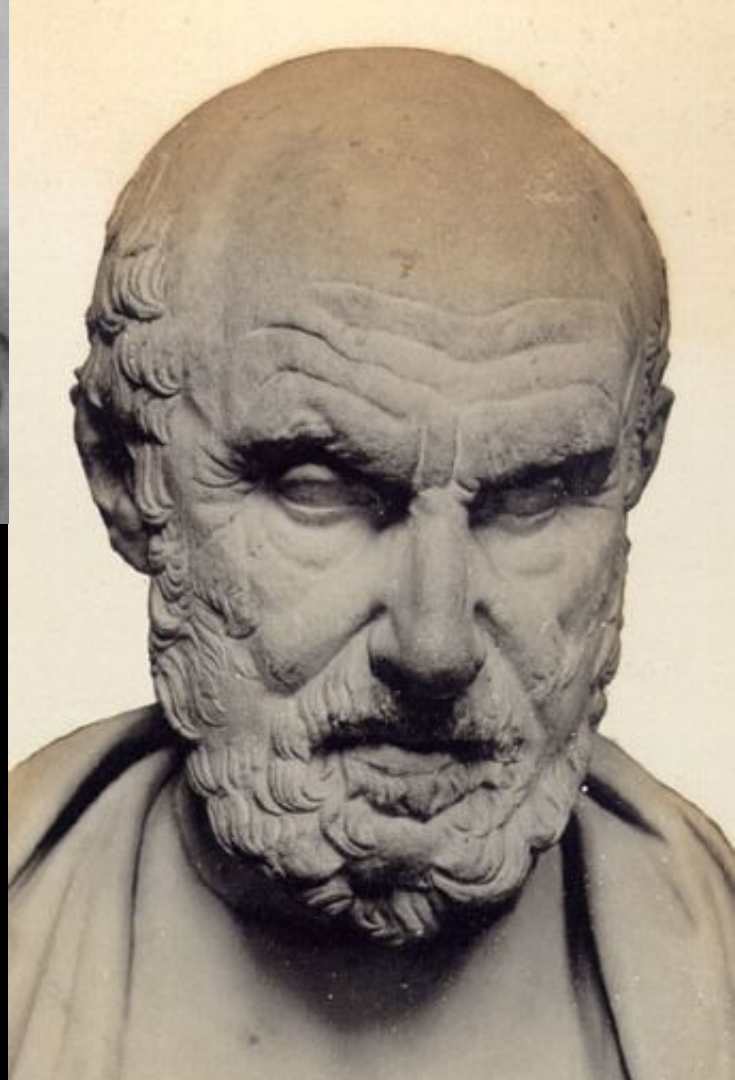
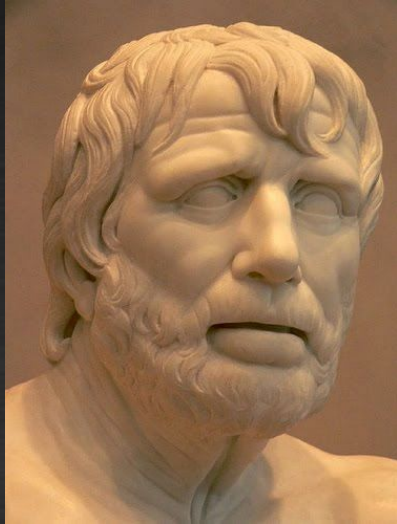
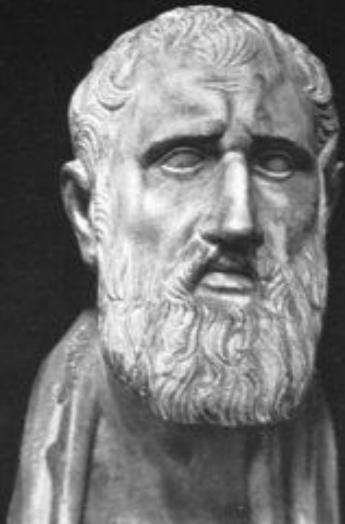
1. unify categorical and truth-functional logic.
2. develop a mechanism by which to distinguish between the Aristotelian and Boolean viewpoint.
3. discover whether Logicism is true.

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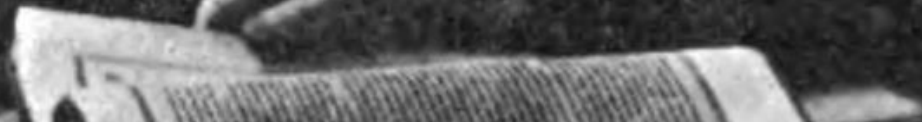
**Aristotle**





# Stoic School of Logic

Frege





**Predicate Logic:  
Important Concepts**



PL

PL (which stands for *predicate language*) is a formal language that mirrors the use of singular terms, predicates, and quantity expressions in natural languages (like English).

# Syntax of PL—Vocabulary

Sentence Letters:  $A, B, C, \dots, Z, A_1, B_1, C_1, \dots, Z_1, \dots$

Predicates:  $A', B', C', \dots, Z', A'_1, B'_1, C'_1, \dots, Z'_1, \dots$

Individual terms:  $a, b, c, \dots, v, a_1, b_1, c_1, \dots, v_1, \dots$

Individual variables:  $w, x, y, z, w_1, x_1, y_1, z_1, \dots$

Truth-functional Connectives:  $\sim, \&, \vee, \supset, \equiv$

Grouping Indicators:  $( ), [ ]$

# Quantifiers:

$(\forall x)$

$(\exists x)$

# Quantifier of PL

A **quantifier of PL** is an expression of PL of the form  $(\forall \mathbf{x})$  or  $(\exists \mathbf{x})$ . An expression of the first form is a *universal* quantifier, and one of the second form is an *existential* quantifier.

# Atomic Formulas of PL

An **atomic formula of PL** is any expression of PL that is either a sentence letter of PL or an **n**-place predicate of PL followed by **n** individual terms of PL.



## Syntax of PL—Rules of Grammar

1. Every atomic formula of PL is a formula of PL.
2. If  $P$  is a formula of PL, so is  $\sim P$ .
3. If  $P$  and  $Q$  are formulas of PL, so are  $(P \ \& \ Q)$ ,  $(P \ \vee \ Q)$ ,  $(P \ \supset \ Q)$ , and  $(P \ \equiv \ Q)$ .
4. If  $P$  is a formula of PL that contains at least one occurrence of  $x$  and no  $x$ -quantifier, then  $(\forall x)P$  and  $(\exists x)P$  are both formulas of PL.

DEF: a **singular term** is a word or phrase that denotes (i.e., refers to) one specifically identified thing.

###

e.g., the following singular terms are proper names:

“Franklin Roosevelt”

“Mount Rushmore”

“Sacramento”

###

###

more examples... the following singular terms  
are definite descriptions:

‘32nd president of the United States’

‘The first person to walk on the moon’

###

DEF: a **general term** is a word or phrase that represents an attribute (i.e., a quality or property) that can be said of more than one thing;

# e.g., “yellow,” “shorter than 6 foot,”

# “happy,” etc.



DEF: an **open sentence** is a predicate expression alone, with a blank attached to it where the subject would be.

# e.g.,

# “\_\_\_\_\_ is yellow.”

# “\_\_\_\_\_ has more than 90 floors.”

DEF: an **individual constant** is a lowercase letter ( $a-v$ ,  $a_1-v_1, \dots$ ) standing for a singular term.

# e.g.,

# a: Adam

# t: tallest building in Chicago

# m: Bob Marley's first guitar

# g: García

DEF: a **predicate letter** is a capital letter  
(A'-Z', A<sub>1</sub>'-Z<sub>1</sub>',...) standing for a particular  
predicate expression.

# e.g.,

# “\_\_\_\_\_ is yellow” can be symbolized as Yx.

DEF: a **variable** is a symbol ( $w-z$ ,  $w_1-z_1, \dots$ ) standing for anything from a given domain (i.e., a group of things.) These stand for singular terms.

# e.g.,

# “x is yellow.”

# “y has more than 90 floors.”

# symbolizing atomic sentences

DEF: an **atomic sentence** is a sentence with a singular term as a subject and one (or more) general term(s) in the predicate.

# e.g.,

# “The tallest building in  
# Chicago has more than 90  
# floors.”



# symbolizing atomic sentences

1. replace the subject part with an individual constant.
2. replace the predicate part with a predicate letter.

RULE: the predicate letter is placed to the left of the individual constant(s).

for example...

“Steve has a backpack.”

Bs

“Mariela is under 6 foot.”

Um

“Pat is a pro golfer.”

Gp

# question:

how would you symbolize  
“Yuri is Russian”?

Ry?

SyntaxError! ‘y’ is not a  
symbol that can stand for  
the singular term “Yuri.”

# we need an individual

# constant (e.g., a-v).

# symbolize this sentence

# instead as:

Ru



# Universal Quantifier

$(\forall x)$

$(\forall x)$  should be  
understood as:

“For all  $x$ ...”

“Everything is good”  
is symbolized as:

$(\forall x) Gx$

# Universal Quantifier

$(\forall x)$

This is read as “Every  $x$  is such that  $x$  is  $G$ ” or “For all  $x$ ,  $x$  is  $G$ ” where  $G$  is “good.”

It literally says that the predicate “\_\_\_\_\_ is good” is true of everything in the universe.

Eg

“Everything is not good.”

$$(\forall x)\sim Gx$$

“All dogs have fleas.”

$$(\forall x)(Dx \supset Fx)$$

# Existential Quantifier

$(\exists x)$

$(\exists x)$  should be read as:

“There exists at least one  $x$  such that...”

“Some things are green”  
is symbolized as:

$(\exists x) Gx$



# Existential Quantifier

$(\exists x)$

This is read as “There exists at least one  $x$  such that  $x$  is  $G$ .”

It literally states that there is at least one thing in the universe of which “\_\_\_\_\_ is green” is true.

Eg

“Some cars are  
noisy”

$$(\exists x)(Cx \ \& \ Nx)$$

“Some students do  
not do their  
homework”

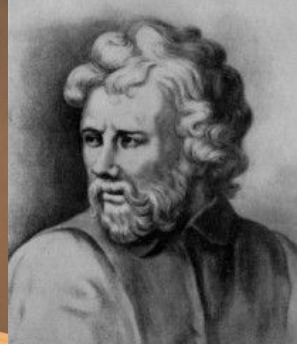
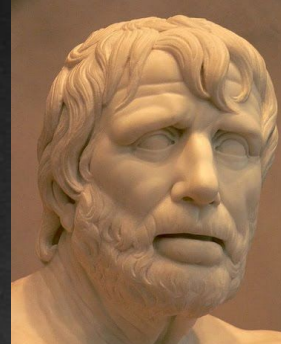
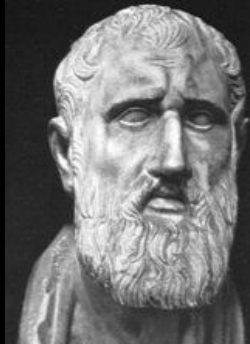
$$(\exists x)(Sx \ \& \ \sim Hx)$$

The following practice problems are taken from Herrick (2013; p. 485):

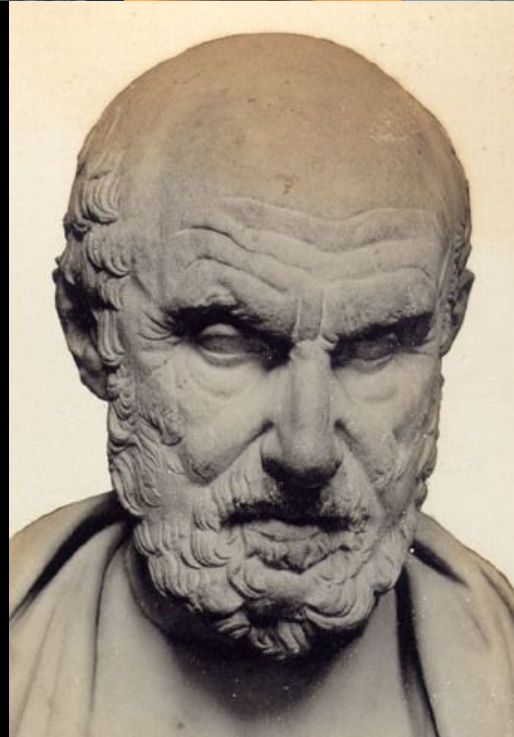
1. All human beings are intrinsically valuable creatures.
2. Some human beings are interested in poetry and some human beings are not.
3. All things that are made of matter are tangible.
4. All visible things are material things.
5. All persons are immaterial souls animating material bodies.
6. All atoms are cloud-like entities.
7. Every person possess dignity and equal moral worth.
8. Every human being deserves to be treated respectfully.



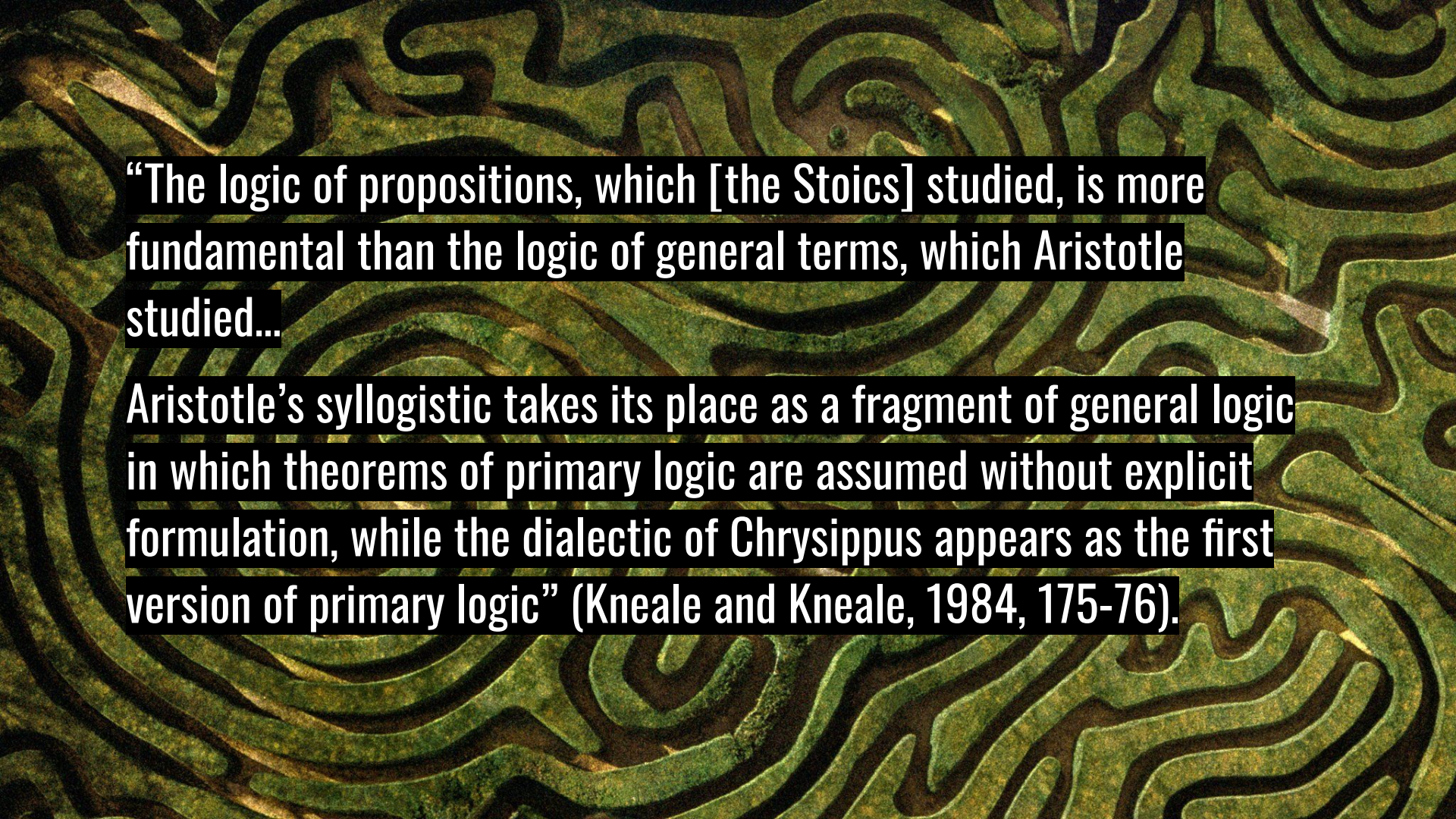
*Food for thought...*



VS





A complex maze with green and brown paths, serving as a background for the text.

**“The logic of propositions, which [the Stoics] studied, is more fundamental than the logic of general terms, which Aristotle studied...**

**Aristotle’s syllogistic takes its place as a fragment of general logic in which theorems of primary logic are assumed without explicit formulation, while the dialectic of Chrysippus appears as the first version of primary logic” (Kneale and Kneale, 1984, 175-76).**

question:

can categorical

propositions be

expressed within PL?

# Remember... ?

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ie, make sure the sentence:

1. begins with a quantifier;
2. the subject term appears second and that it designates a class of things;
3. the copula appears next; and
4. the predicate term appears last, and it also designates a class of things.



“All dogs are animals.”

$$\forall (x) (Dx \supset Ax)$$

# note:

# any wff of TL counts as a wff of PL, but

# not every wff of PL counts as a wff of TL.

# PL, then, is said to “contain” TL.

# in other words, Stoic logic is contained

# within PL.

###

So, categorical propositions and any wff of TL are expressible in PL.

Hence, Aristotle's categorical logic and Stoic truth-functional logic are both "contained" within PL.

The two forms of logic have been unified;)

###

Homework!  
Read and re-do in class  
practice problems!

NOT  
Sorry