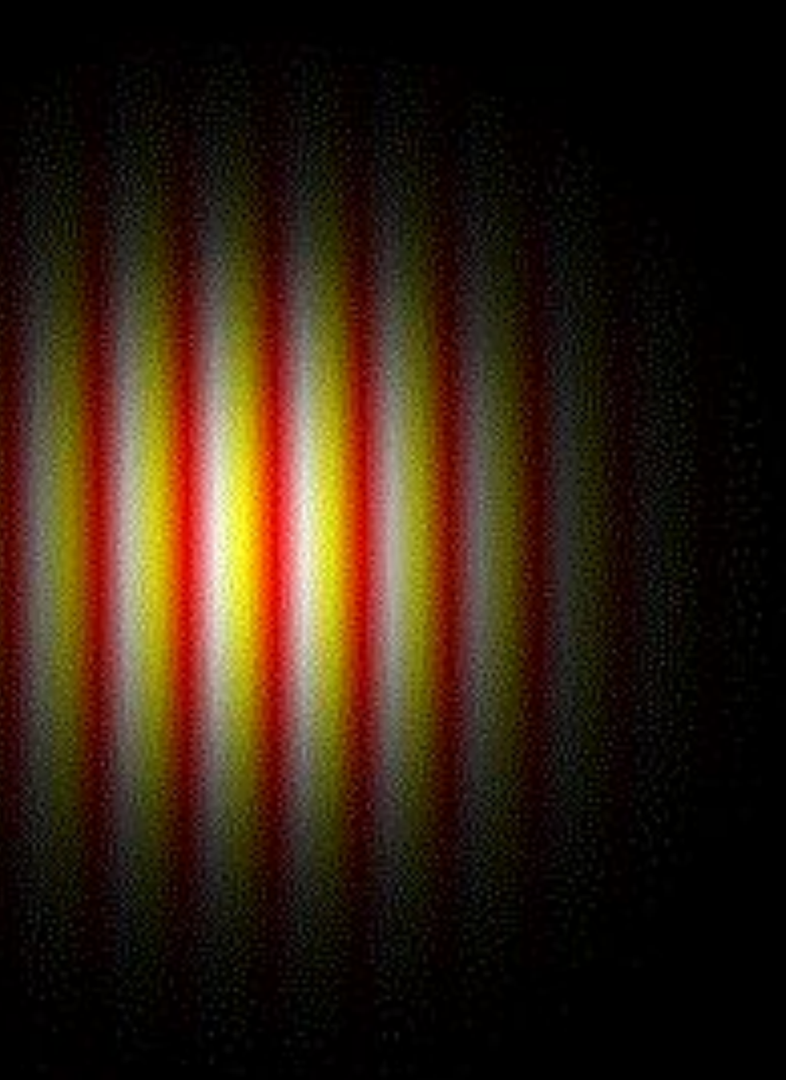
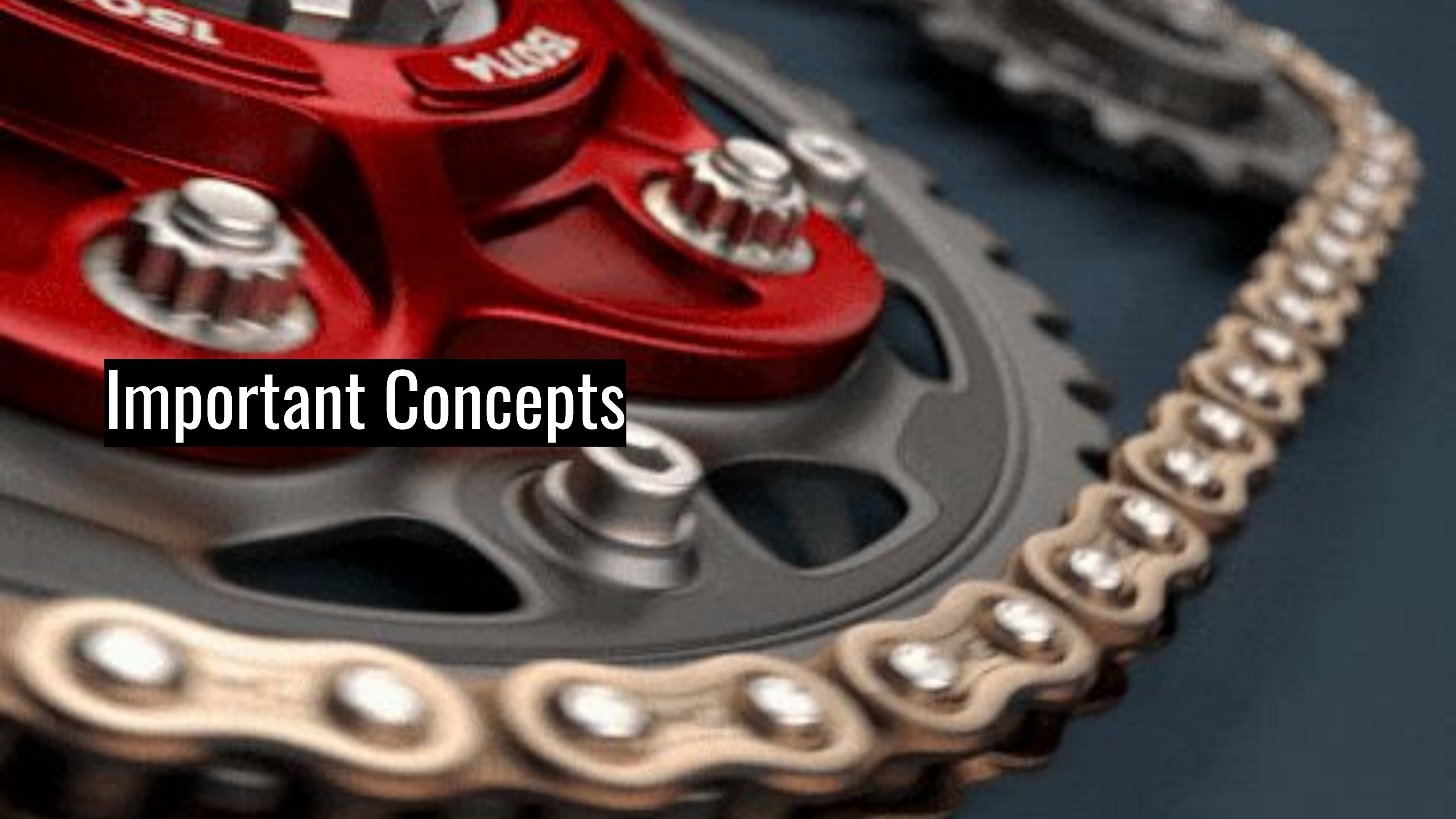


Quantifiers

Important Concepts



Logical operator

An expression of PL that is either a quantifier or a truth-functional connective is a logical operator of PL.

Scope

In a sentence containing just one quantifier, the scope of a quantifier is the quantifier itself plus the expression enclosed in parentheses to the immediate right of the quantifier.

Bound variable

A bound variable is any occurrence of a variable **x** in a formula **P** of PL that is within the scope of an **x**-quantifier.

E.g., $(\exists x)(Nx \,\&\, Cx)$

Free variable

A free variable is any occurrence of a variable **x** in a formula **P** of PL that is not bound.

E.g., Nx & Cx

Sentence of PL

A formula **P** of PL is a sentence of PL if and only if no occurrence of a variable in P is free.

In other words, "($\exists$x)(Nx & Cx)" is a sentence of PL; "(Nx & Cx)" is not a sentence of PL.

Practice: TLB, p. 275 #2.

Translations

Universal Affirmative

All _____ are _____.

1. Whales are mammals.
2. Any whale is a mammal.
3. A whale is a mammal.
4. Every whale is a mammal.

$$(\forall x)(Wx \supset Mx)$$

Universal Negative

No _____ are _____.

1. Whales are not reptiles.
2. A whale is not a reptile.
3. Every whale is a non-reptile.

$$(\forall x)(Wx \supset {\sim}Rx)$$

Existential Affirmative

Some _____ are _____.

Some trees are poplars.

$(\exists x)(Tx \ \& \ Px)$

Question:
Why not $(\exists x)(Tx \supset Px)$?

This reads: "There exists an x such that if x is a tree, then x is a poplar.

Existential Negative

Some _____ are not _____.

Some students are not Republicans.

$$(\exists x)(Sx \ \& \ {\sim}Rx)$$

The Four Sentences of Categorical Logic

The universal affirmative (A): All F are G

The universal negative (E): No F are G

The particular affirmative (I): Some F are G

The particular negative (O): Some F are not G

The universal affirmative (A): $(\forall \mathbf{x})(\mathbf{P} \supset \mathbf{Q})$

The universal negative (E): $(\forall \mathbf{x})(\mathbf{P} \supset \sim\mathbf{Q})$

The particular affirmative (I): $(\exists \mathbf{x})(\mathbf{P} \ \& \ \mathbf{Q})$

The particular negative (O): $(\exists \mathbf{x})(\mathbf{P} \ \& \ \sim\mathbf{Q})$

The universal affirmative (A): $(\forall x)(Fx \supset Gx)$

The universal negative (E): $(\forall x)(Fx \supset \sim Gx)$

The particular affirmative (I): $(\exists x)(Fx \,\&\, Gx)$

The particular negative (O): $(\exists x)(Fx \,\&\, \sim Gx)$

Translating sentences with multiple adjectives

To accurately convey the meaning behind statements with multiple adjectives, use multiple predicate constants.

"All friendly old cats purr"

$(\forall x)((Fx \& (Ox \& Cx)) \supset Px)$

"All old werewolves are gruesome"

$(\forall x)((Ox \& Wx) \supset Gx)$

Symbolizing existence

How would you symbolize the following?

“Nessie exists”

En?

Nope. Existence is not a predicate.

The correct way to symbolize is:

$(\exists x)Nx$

“Santa Claus does not exist” is

$\sim(\exists x)Sx$

More examples

Translate the following:
"Bats and rats are mammals"

$(\forall x)((Bx \vee Rx) \supset Mx)$

Translate the following:
"Only teachers with certification were hired"

Paraphrase:
For any x, if x was hired, then x was a teacher and x had a certification.

$$(\forall x)(Hx \supset (Tx \,\&\, Cx))$$

Translate the following:
“Any person who likes ‘The Cable Guy’ has a dark sense of humor”

Paraphrase:
For all x, if x is a person and x likes “The Cable Guy”, then x has a dark sense of humor.

$(\forall x)((Px \ \& \ Cx) \supset Dx)$

Swapping quantifiers

"Everything is good" is logically equivalent to "There is nothing that is non-good."

Hence, it can be symbolized in two ways:

$$(\forall x)\ Gx$$

or

$$\sim(\exists x)\ \sim Gx$$

Practice: TLB, p. 294 #1.

Homework!

Read Chapter 7 (p. 276-94) of TLB!