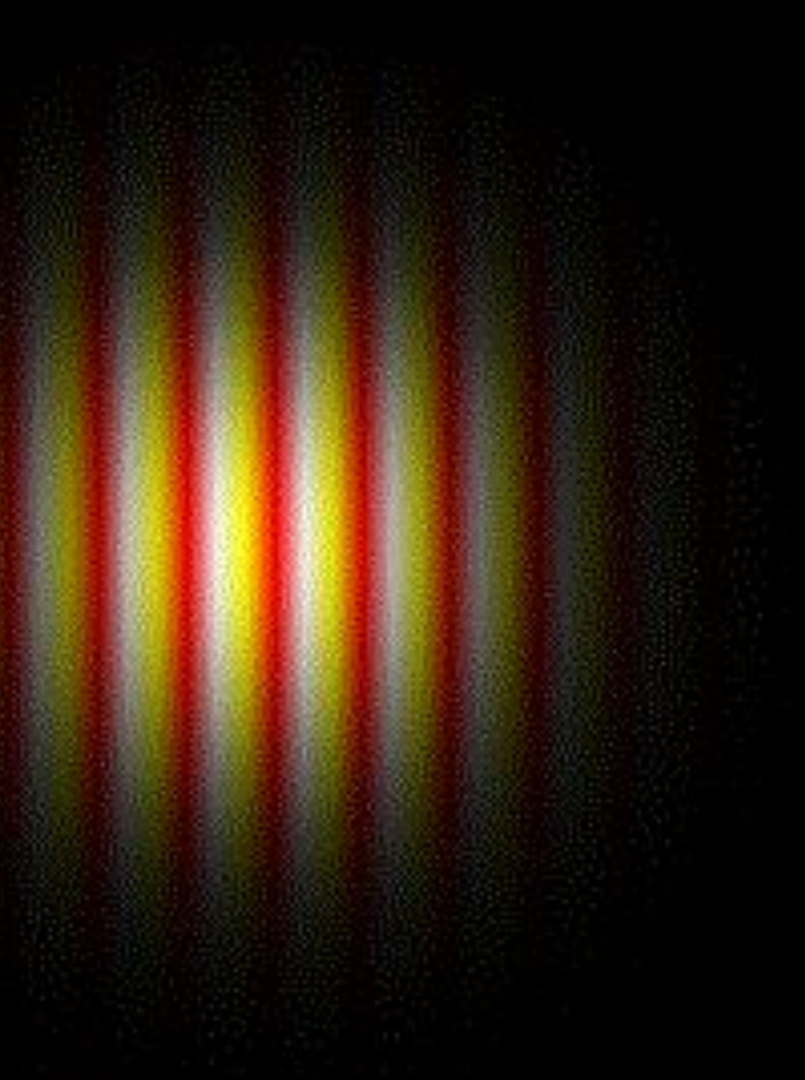


Categorical Logic 1.0

Homework!
Memorize this!

Logically True Sentences

- Either Yuri is a tricycle or Yuri is not a tricycle.
- It’s false that I am a banana and I am not a banana.

Logically False Sentences

- I am a banana and I am not a banana.

Logically Indeterminate Sentences

- If you leave by 8, you'll show up to school on time.

Logical Entailment

{“Ann likes swimming”,

“Bob likes pudding”,

“Carlos hates Dan”}

$\models$

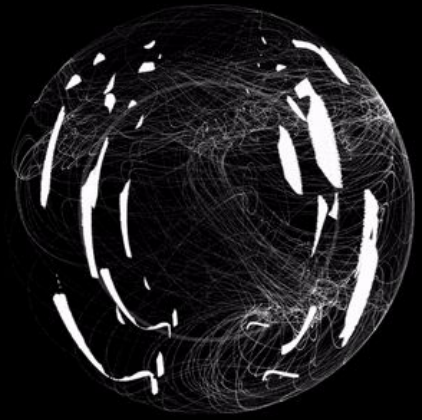
“Carlos hates Dan”

___

Special cases of logical concepts

1. An argument with a conclusion that is logically true is logically valid no matter what premise(s) it has.

literally any set of sentences



Therefore, either I am a burrito or I am not.

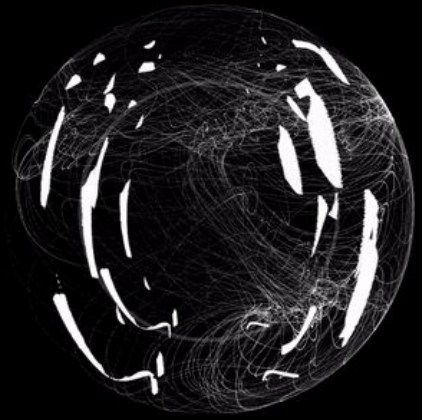
Special cases of logical concepts

1. An argument with a conclusion that is logically true is logically valid no matter what premise(s) it has.
2. Any set of sentences entails every logically true sentence.

literally any set of sentences

$\models$

Therefore, either I am a burrito or I am not.



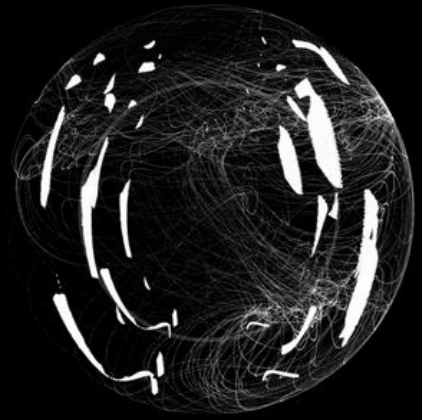
Special cases of logical concepts

1. An argument with a conclusion that is logically true is logically valid no matter what premise(s) it has.
2. Any set of sentences entails every logically true sentence.
3. An argument with premises that form an inconsistent set is always logically valid.

1. All oscillators are truculent.
2. Some oscillators are not truculent.

---

3. Therefore, penguins love kittens.



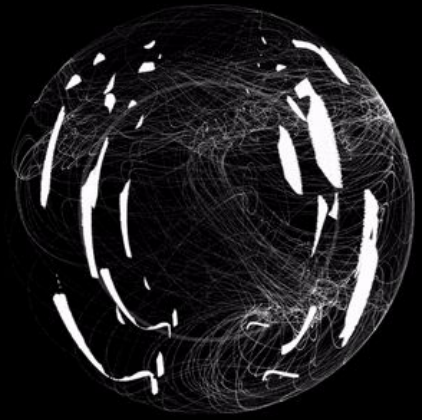
Special cases of logical concepts

1. An argument with a conclusion that is logically true is logically valid no matter what premise(s) it has.
2. Any set of sentences entails every logically true sentence.
3. An argument with premises that form an inconsistent set is always logically valid.
   Note: This includes any argument with a logically false premise.

1. I am a burrito and I am not a burrito.

---

2. Therefore, penguins love kittens.



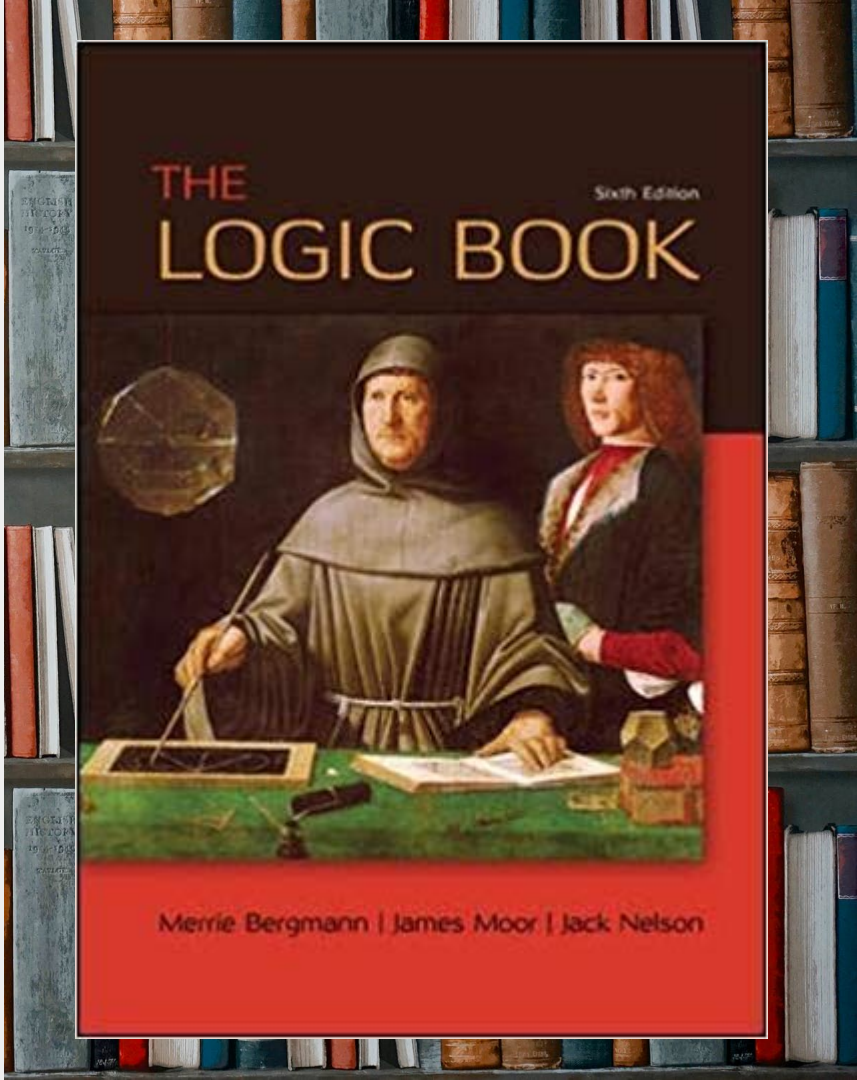
Special cases of logical concepts

1. An argument with a conclusion that is logically true is logically valid no matter what premise(s) it has.
2. Any set of sentences entails every logically true sentence.
3. An argument with premises that form an inconsistent set is always logically valid.
   Note: This includes any argument with a logically false premise.
4. All logically true sentences are logically equivalent.
5. All logically false sentences are logically equivalent.

Why is this the case?

It will be easier to understand this once we take a look at truth-tables and truth-functions in Unit II.

The interested student can also read Chapter 1, Section 3 of *The Logic Book*.



Question:
How do we evaluate inductive arguments?

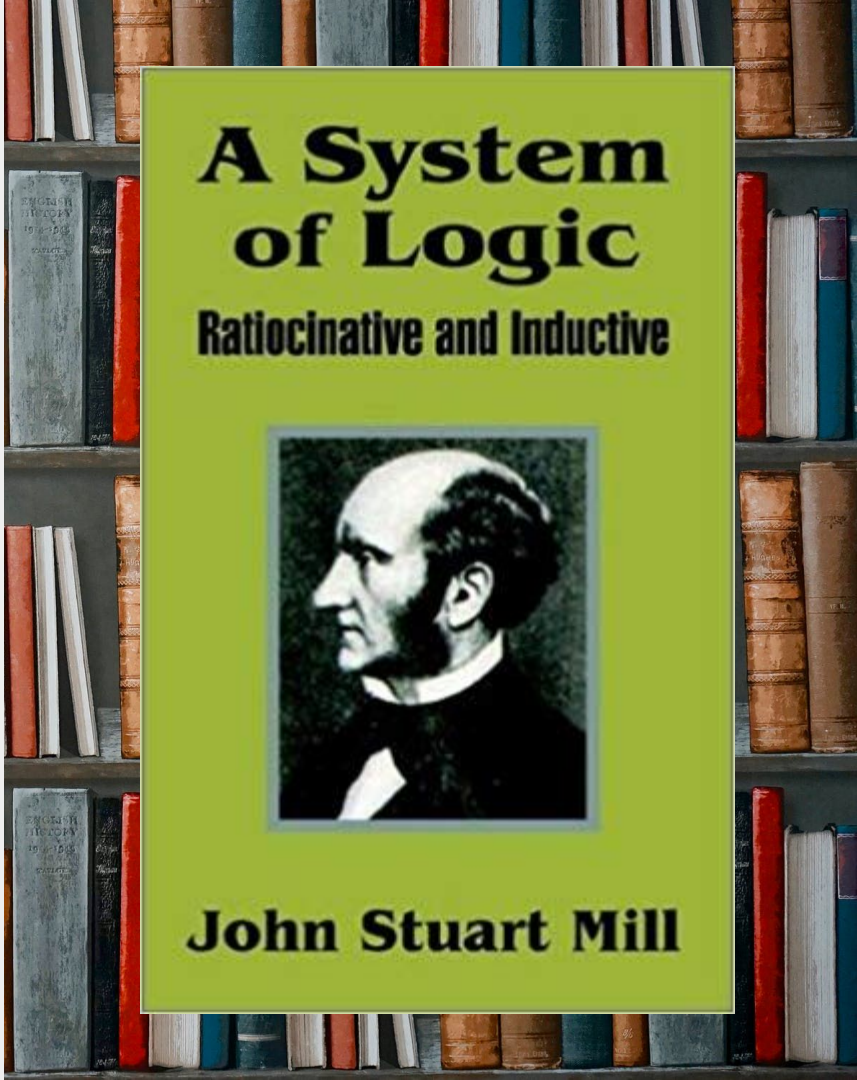
An inductive argument...

...is said to be strong when, if the premises are true, then the conclusion is very likely true.

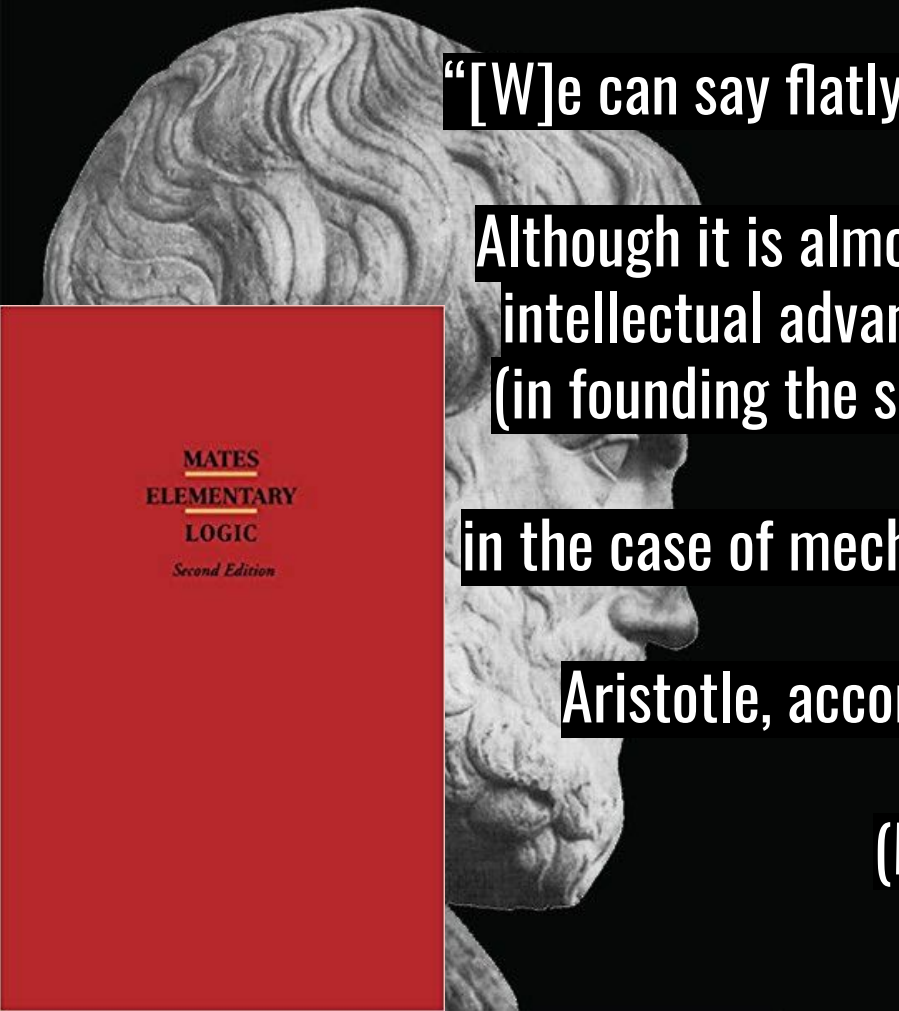
...is cogent when a. it is strong; and b. it has true premises.

During the middle of the 19th century there were great advancements in formal deductive logic (which will be covered in this course), as well as a revival in inductive logic spearheaded by the British philosopher John Stuart Mill, who is mostly known for his Utilitarian system of ethics.

Inductive logic will unfortunately not be covered in this course, but Mill's collected works are available online (see also Hurley 1985, chapter 9).

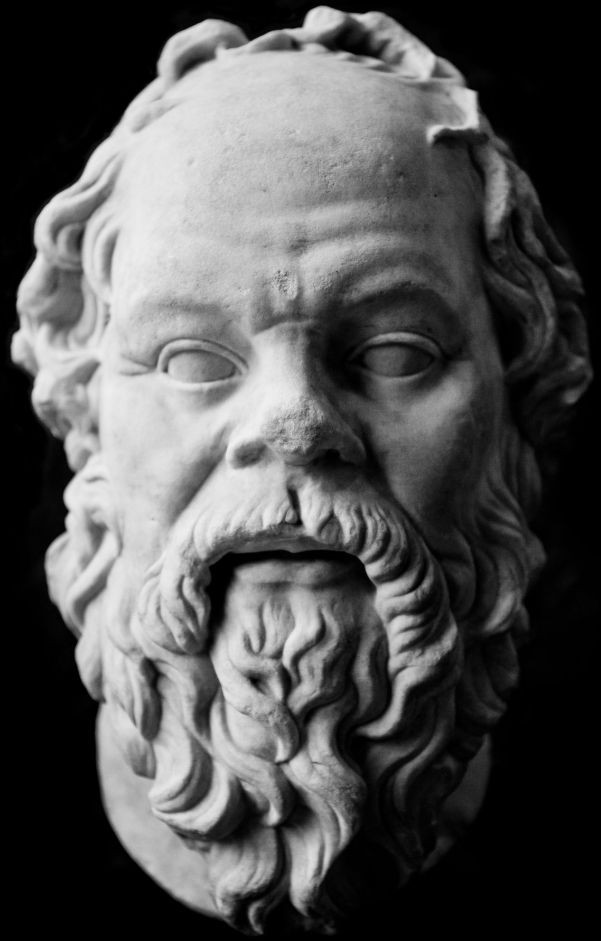


Storytime!



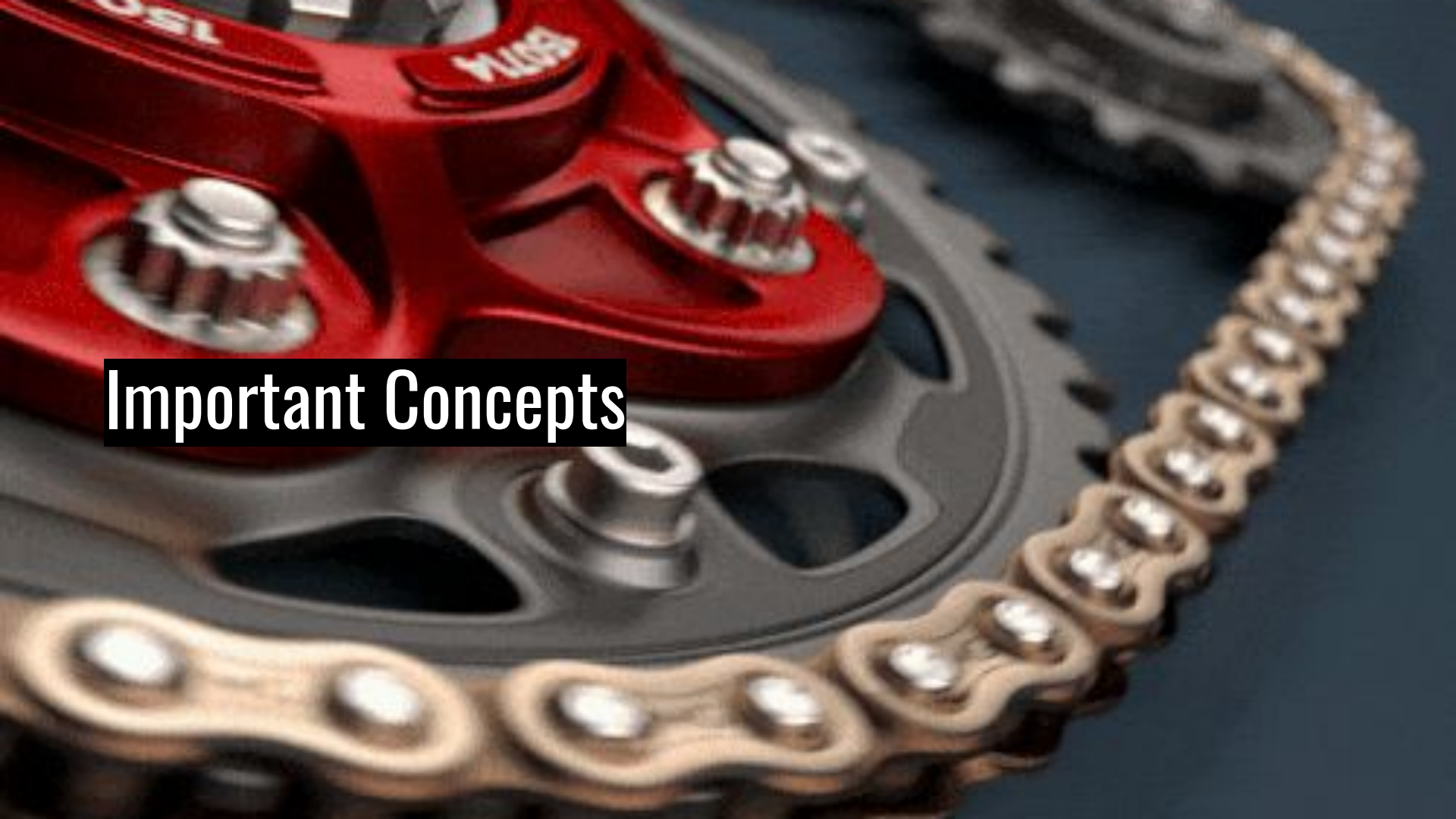
"[W]e can say flatly that the history of [Western] logic begins with the Greek Philosopher Aristotle... Although it is almost a platitude among historians that great intellectual advances are never the work of only one person (in founding the science of geometry Euclid made use of the results of Eudoxus and others; in the case of mechanics Newton stood upon the shoulders of Descartes, Galileo, and Kepler; and so on), Aristotle, according to all available evidence, created the science of logic absolutely ex nihilo" (Mates 1972: 206; interpolations are mine).

The Socratic Elenchus



Sample Dialogue

S: Yo, so you believe in X, right?
D: Ya
S: And you also believe in Y, no?
D: Shizzle
S: But X and Y form a logically inconsistent set. In other words, they can’t both be true.
D: Noooo…



Important Concepts

Categorical logic is the study of arguments composed of sentences that state relationships between categories.

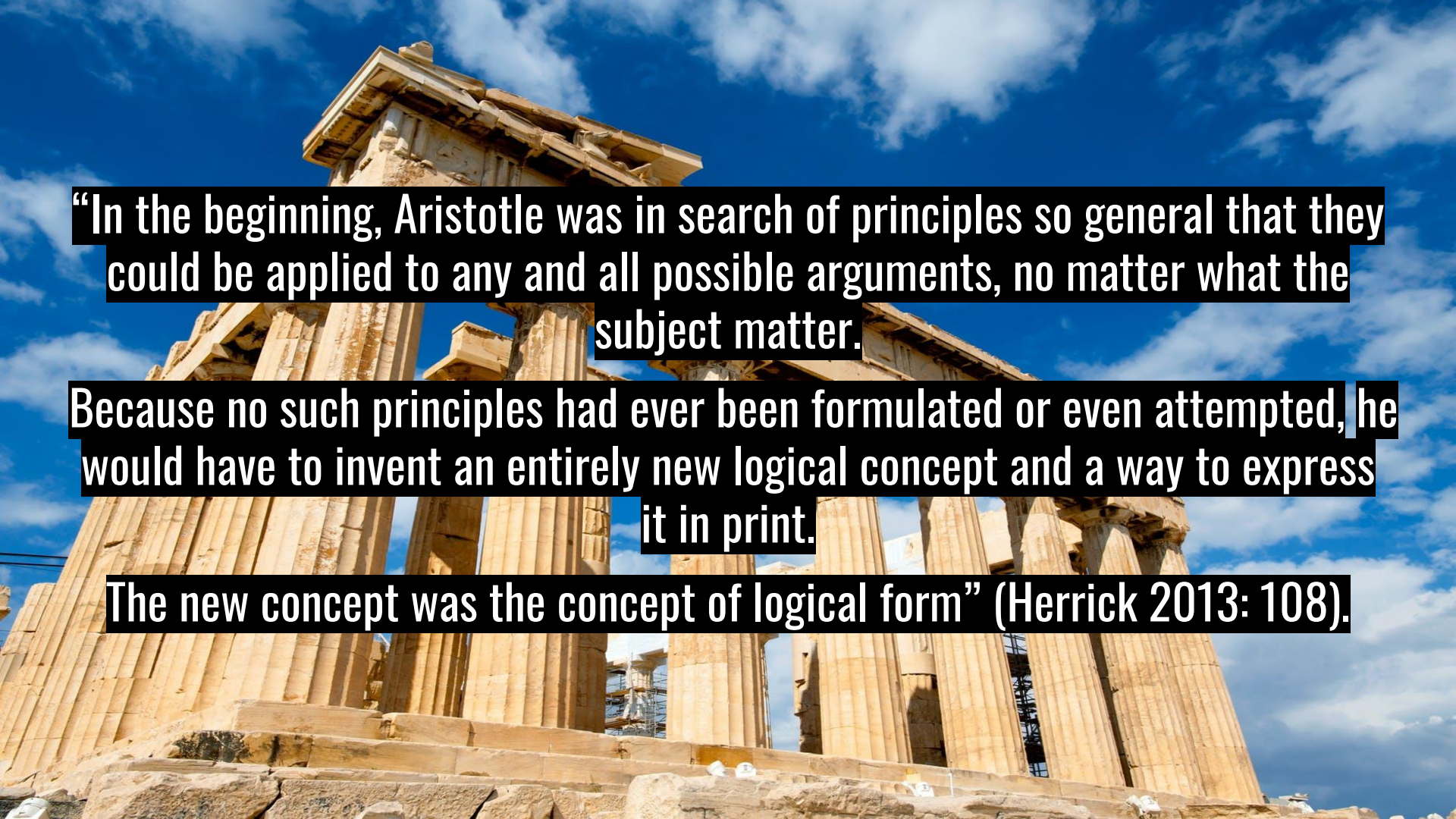
A class is a group of things that have a specified characteristic in common, eg cats, flowers, etc.

A quantifier is a word that tells us the quantity of things in the subject category that are said to belong to the predicate category.

The copula is a word that joins the subject to the predicate, ie “are”, “are not”.

Example!

All dogs are mammals.



“In the beginning, Aristotle was in search of principles so general that they could be applied to any and all possible arguments, no matter what the subject matter.

Because no such principles had ever been formulated or even attempted, he would have to invent an entirely new logical concept and a way to express it in print.

The new concept was the concept of logical form” (Herrick 2013: 108).

Consider the following sentences:

1. All dogs are mammals.
2. All snakes are reptiles.
3. All robins are birds.

Although each is about a different subject matter, these sentences all share a common structure:
All **S** are **P**.
This common structure is called the **logical form** of the three sentences, and the three sentences are called *substitution instances* of that form.

The Four Sentences of Categorical Logic

The universal affirmative (A): All **S** are **P**
The universal negative (E): No **S** are **P**
The particular affirmative (I): Some **S** are **P**
The particular negative (O): Some **S** are not **P**

Note:

Logicians define the word “some” as “at least one.”

Standard Form in Categorical Logic

Make sure that the premises/conclusion are in the following order...

1. Quantifier
2. Subject class (led by a noun)
3. Copula
4. Predicate class (led by a noun)

Sentences with non-standard predicates

“Some aardvarks are cute.”

Some aardvarks are animals that are cute.

“All cars are metallic.”

All cars are things that are metallic.

Sentences with non-standard quantifiers

“A tiger is a mammal.”
All tigers are mammals.

“A whale is a beautiful creature.”
All whales are creatures that are beautiful.

Sentences with missing copulas

“All ducks swim.”
All ducks are animals that swim.

“Some birds fly south for the winter.”
Some birds are animals that fly south for the winter.

Sentences not in the present tense

“Some democrats will be elected.””

Some democrats are persons who will be elected.

Sentences with proper names

"Aristotle is a logician."

All persons identical to Aristotle are persons who are logicians.

Sentences with adverbs

“Aristotle always paces back and forth when he lectures.”

All persons identical to Aristotle are persons who pace back and forth while they lecture.

“Plato is never unreasonable.”

No persons identical to Plato are persons are unreasonable.

Conditional sentences

“If it is a mouse, then it is a mammal.”
All mice are mammals.

“If it is a mouse, then it is not a reptile.”
No mice are reptiles.

Exceptive statements

"All except truckers are happy with the new regulations."

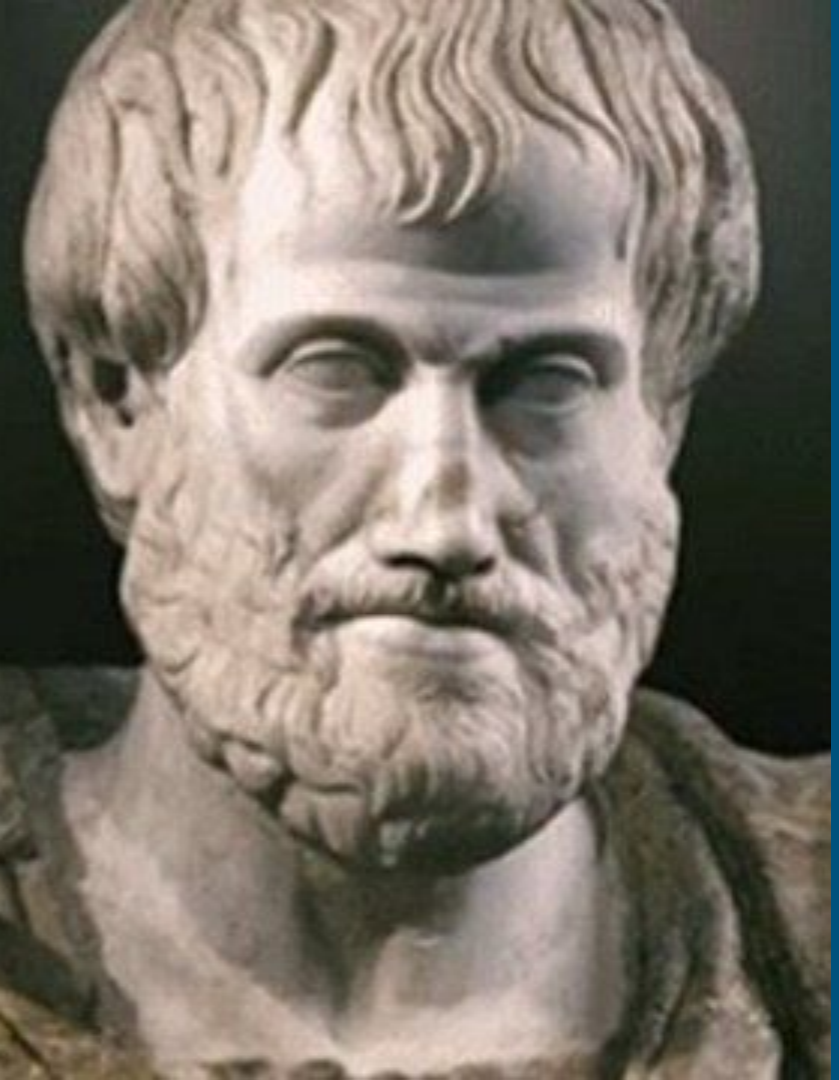
All non-truckers are persons who are happy with the new regulations;

No truckers are persons who are happy with the new regulations.

Three Types of Categorical Reasoning

An immediate inference is an argument composed of exactly one premise and one conclusion immediately drawn from it.
E.g.,

1. All Spartans are brave persons.
2. So, some brave persons are Spartans.

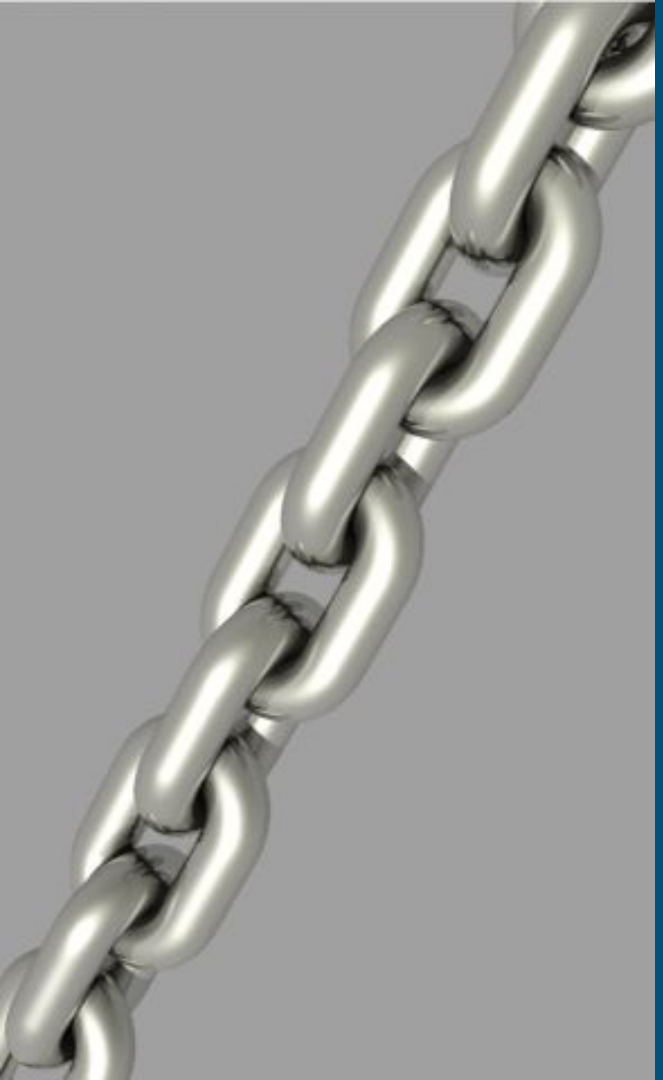


A mediate inference is an argument composed of exactly two premises and exactly one conclusion in which the reasoning from the first premise to the conclusion is “mediate” by passing through a second premise.

This type of argument is also called a “categorical syllogism.”

E.g.,

1. All goats are mammals.
2. All mammals are animals.
3. Therefore, all goats are animals.



A sorites is a chain of interlocking mediate inferences leading to one conclusion in the end.

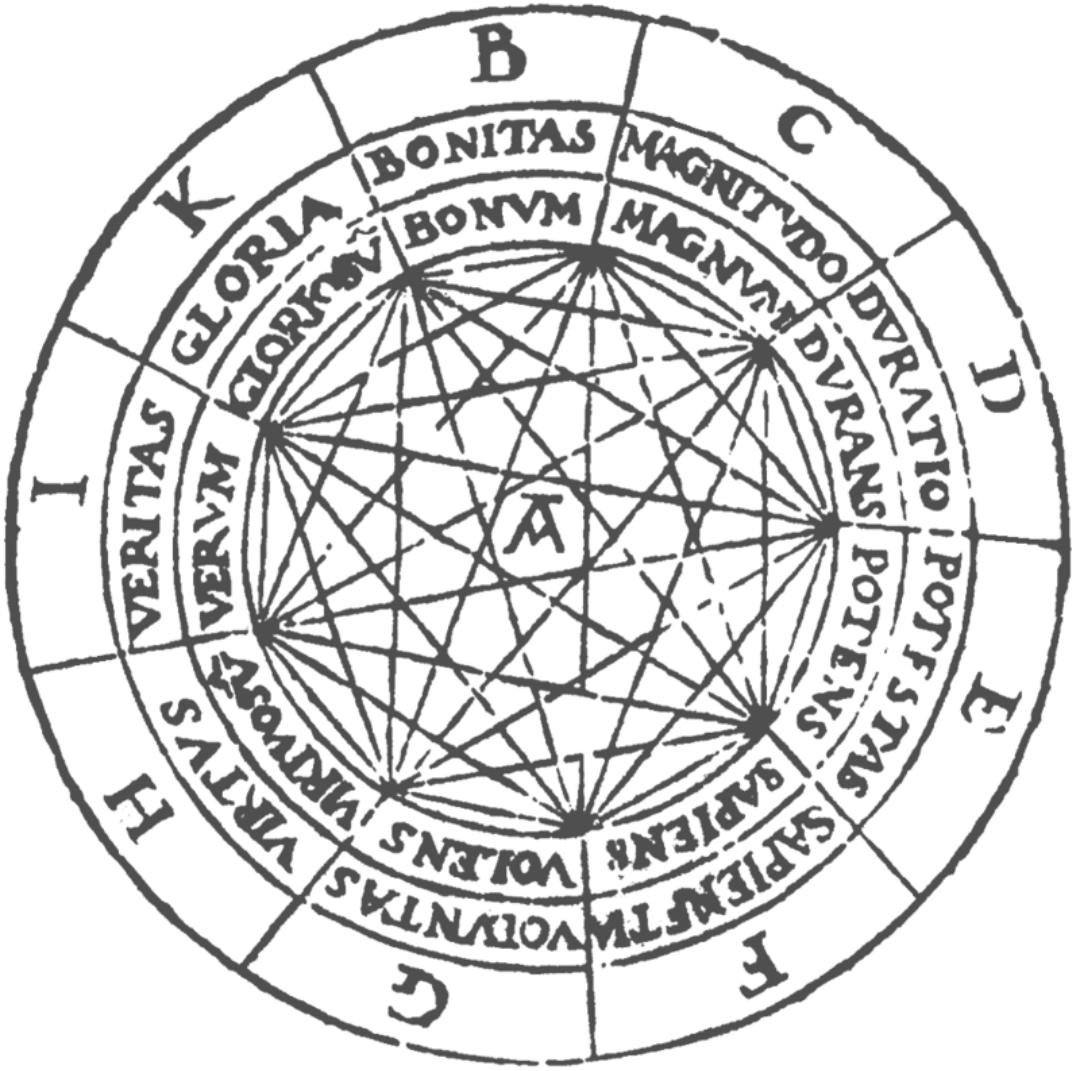
E.g.,

1. All Spartans are warriors.
2. All warriors are brave persons.
3. All brave persons are strong persons.
4. So, all Spartans are strong persons.

Aristotle developed two methods for assessing one-premise arguments, or immediate inferences:

1. The Square of Opposition, and
2. The laws of conversion, obversion, and contraposition.

We will focus on the Square of Opposition.


B
C
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D
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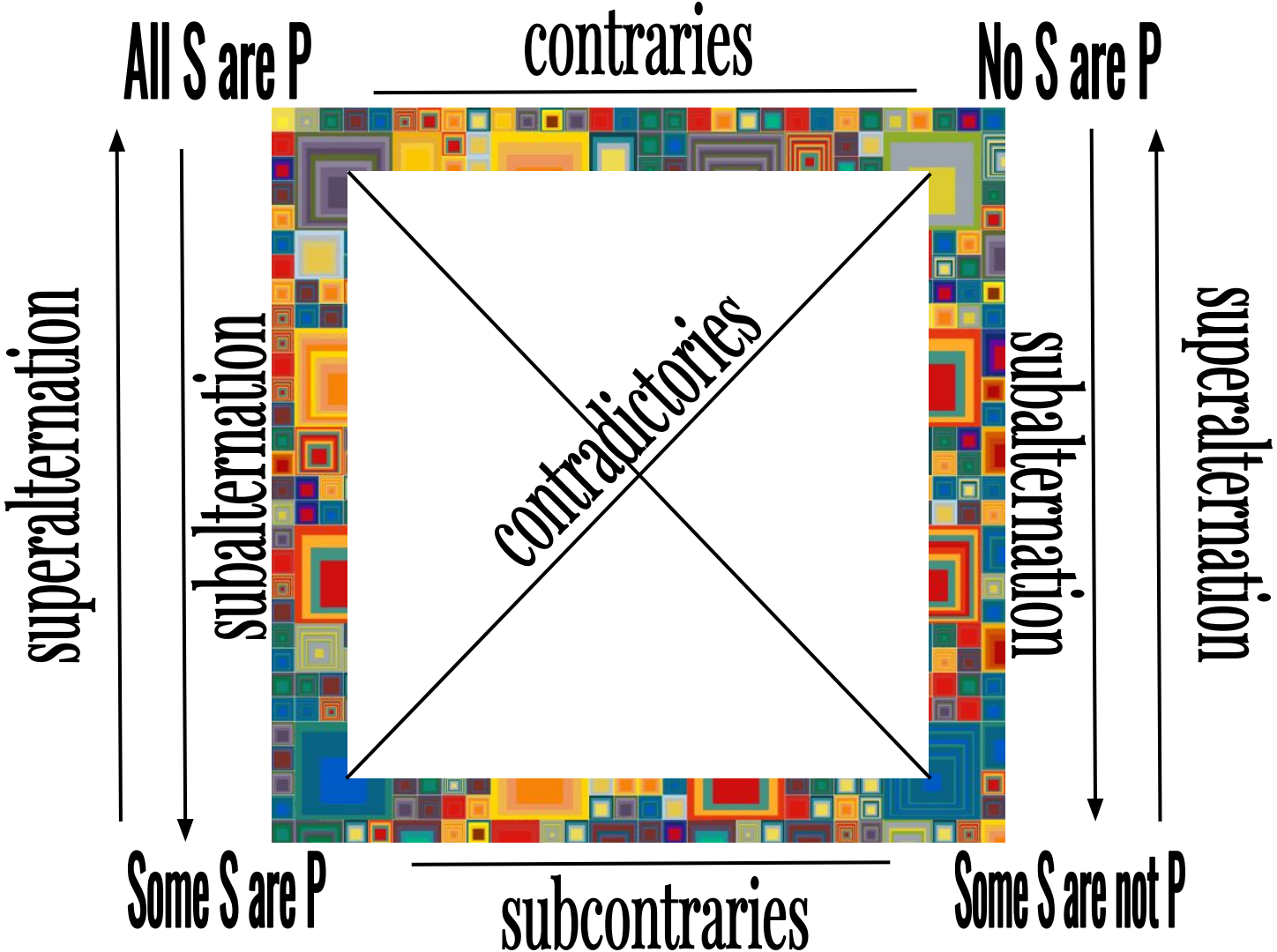
All S are P
contraries
No S are P
superalternation
subalternation
contradictories
subalternation
superalternation
Some S are P
subcontraries
Some S are not P

Homework!

This!