

# Truth-Tables (Pt. I)

# Activity!

## The Replacement Method



Steps:

1. Copy the formula, substituting the letter constants with the relevant truth-values.
2. Calculate the operators with the smallest scopes.
3. Calculate the main operator.

—

Use the replacement method to calculate the truth-values of the following:

(Key: R and L are true; M and O are false.)

a.  $\sim[(O \vee M) \supset (R \equiv M)]$

b.  $[(R \& L) \vee (R \& M)] \& \sim(L \& M)$

c.  $(([R \equiv L] \& [M \equiv O]) \vee [M \supset R]) \supset O$

–

**Question:**  
**How do we know**  
**these argument**  
**forms are *always***  
**valid?**



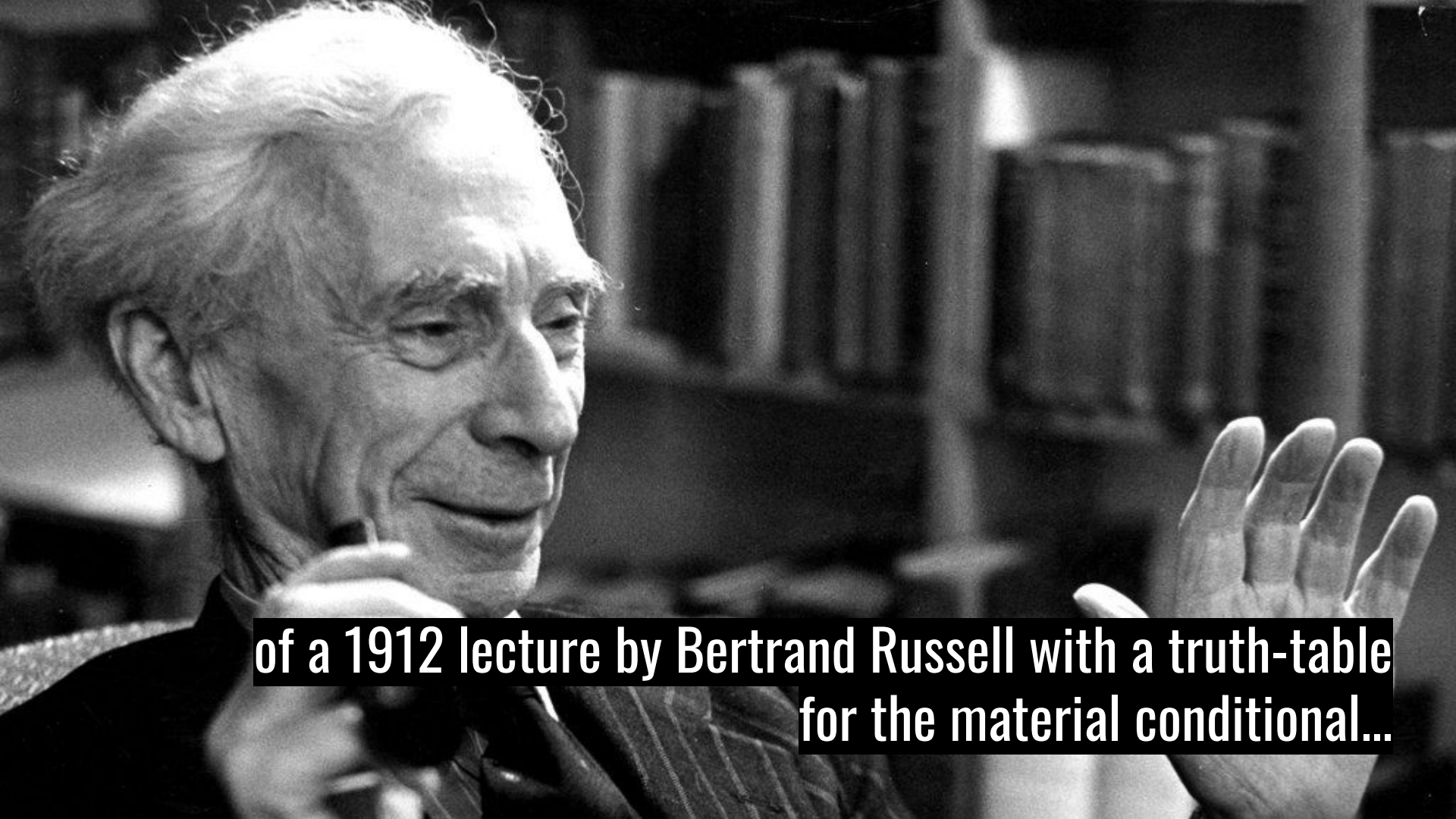
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# Storytime!



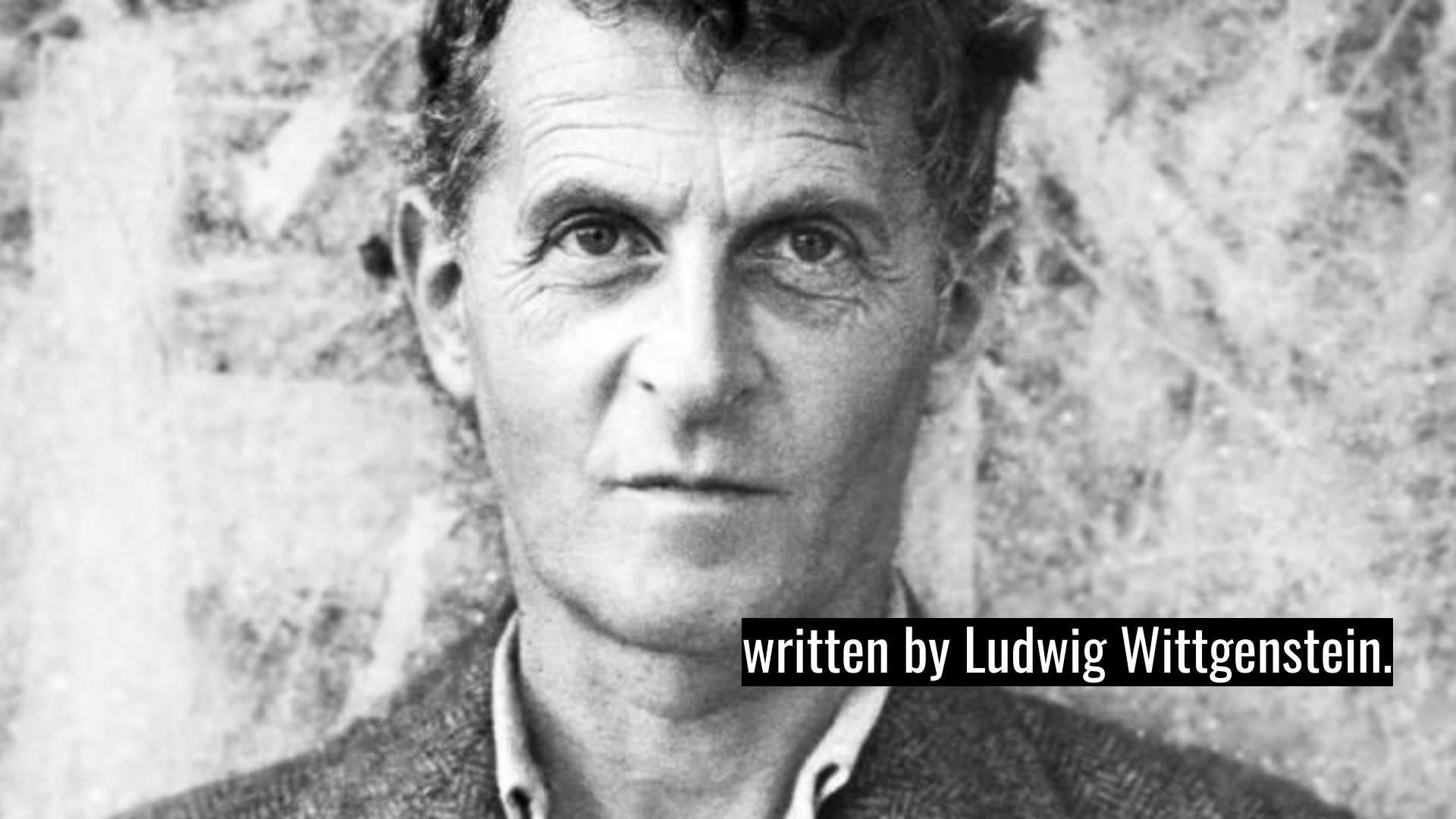


**In 1997, John Shosky discovers a page of a typed transcript...**



**of a 1912 lecture by Bertrand Russell with a truth-table  
for the material conditional...**

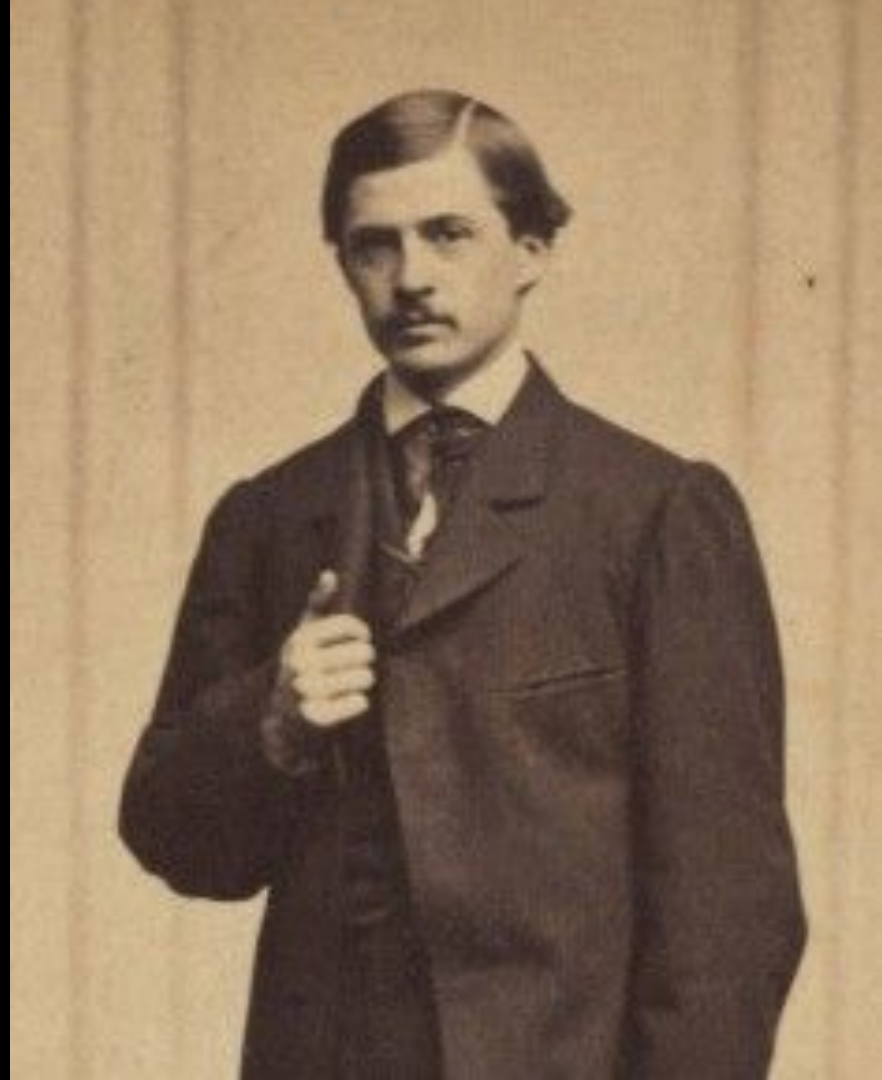




**written by Ludwig Wittgenstein.**

P	Q	P and Q
T	T	T
T	F	F
F	T	F
F	F	F

But subsequent research by Irving Anellis (2012) showed that “an unpublished manuscript identified as composed by [Charles] Peirce in 1893 includes a truth table matrix that is equivalent to the matrix for material implication discovered by John Shosky.



A man with dark hair, wearing dark sunglasses and a black jacket over a maroon shirt, is shown in profile looking to the left. He has a surprised expression. The background is a blurred indoor setting with a stone wall and some plants. The word "WOW" is overlaid in large, bold, yellow letters with a purple shadow effect at the bottom center of the image.

**WOW**



# Truth-table Analysis

...

**SENTENCE EDITION**

# Steps

**#1: Write in the following:**

- **the sentence constants (in alphabetical order)**
- **the TL-symbolization of the sentence**





A

B

(A

&

B)

≡

~

B

## #2: Draw the table.

Note: There should be enough columns for the truth-value assignments and all the sentence constants and connectives of the formula in TL; there should be enough rows for all the possible truth-values of the letter constants.

Rule: If there is one letter constant, there should be three rows; if there are two constants, there should be 5 rows; 3 constants, 9 rows; etc.

(In other words,  $N = 2^x + 1$ , where  $N$  equals the number of rows and  $x$  equals the number of sentence constants.)



A

B

(A

&

B)

≡

~

B



A	B	(A	&	B)	$\equiv$	$\sim$	B



A	B	(A	&	B)	≡	~	B



A	B	(A	&	B)	≡	~	B



A	B	(A	&	B)	≡	~	B

**#3: Fill in all possible truth-value assignments.**

**Rule: If there are two letter constants, input T-T-F-F for the first column then T-F-T-F for the second; if there are three letter constants, input T-T-T-T-F-F-F-F for the first column, then T-T-F-F-T-T-F-F for the second, then T-F-T-F-T-F-T-F for the third.**



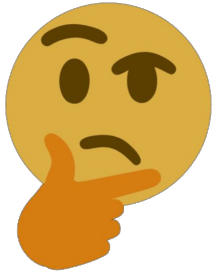


A	B	(A	&	B)	≡	~	B

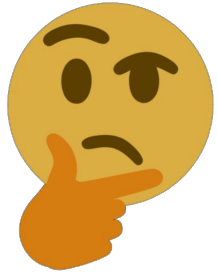


A	B	(A	&	B)	≡	~	B
T	T						
T	F						
F	T						
F	F						





A	B	C	(A	&	C)	≡	~	B
T	T	T						
T	T	F						
T	F	T						
T	F	F						
F	T	T						
F	T	F						
F	F	T						
F	F	F						



A	B	C	(A	&	C)	=	~	B
T	T	T						
T	T	F						
T	F	T						
T	F	F						
F	T	T						
F	T	F						
F	F	T						
F	F	F						



A	B	(A	&	B)	≡	~	B
T	T						
T	F						
F	T						
F	F						



A	B	(A	&	B)	≡	~	B
T	T	T					
T	F	T					
F	T	F					
F	F	F					



A	B	(A	&	B)	≡	~	B
T	T	T		T			T
T	F	T		F			F
F	T	F		T			T
F	F	F		F			F



**#4: Find the main connective.**



A	B	(A	&	B)	≡	~	B
T	T	T		T			T
T	F	T		F			F
F	T	F		T			T
F	F	F		F			F



**#5: Calculate the values under the connectives with the smallest scope.**

**Note: If there is a tie for which is the smallest scope, compute the leftmost operator first.**



A	B	(A	&	B)	≡	~	B
T	T	T		T			T
T	F	T		F			F
F	T	F		T			T
F	F	F		F			F





A	B	(A	&	B)	≡	~	B
T	T	T		T		F	T
T	F	T		F		T	F
F	T	F		T		F	T
F	F	F		F		T	F





A	B	(A	&	B)	≡	~	B
T	T	T	T	T		F	T
T	F	T	F	F		T	F
F	T	F	F	T		F	T
F	F	F	F	F		T	F



**#6: Calculate the value of the main  
connective, i.e., the final column.**



A	B	(A	&	B)	≡	~	B
T	T	T	T	T		F	T
T	F	T	F	F		T	F
F	T	F	F	T		F	T
F	F	F	F	F		T	F







A	B	(A	&	B)	≡	~	B
T	T	<del>T</del>	T	<del>T</del>		F	<del>T</del>
T	F	<del>T</del>	F	<del>F</del>		T	<del>F</del>
F	T	<del>F</del>	F	<del>T</del>		F	<del>T</del>
F	F	<del>F</del>	F	<del>F</del>		T	<del>F</del>





A	B	(A	&	B)	≡	~	B
T	T	<del>T</del>	T	<del>T</del>	F	F	<del>T</del>
T	F	<del>T</del>	F	<del>F</del>	F	T	<del>F</del>
F	T	<del>F</del>	F	<del>T</del>	T	F	<del>T</del>
F	F	<del>F</del>	F	<del>F</del>	F	T	<del>F</del>





A	B	(A	&	B)	≡	~	B
T	T	T	T	T	F	F	T
T	F	T	F	F	F	T	F
F	T	F	F	T	T	F	T
F	F	F	F	F	F	T	F



**#7: Review your work.**



Construct a truth-table to calculate the truth-values of the following formulas:

a.  $\sim[(O \vee P) \supset (O \equiv P)]$

b.  $[K \supset C] \supset A$

c.  $(T \ \& \ L) \vee (T \ \& \ M)$

# Truth-table Analysis

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**ARGUMENT EDITION**

# Steps



**#1: Follow all steps from SENTENCE EDITION.**

## Rule for Validity Test:

If there is any row on the truth-table that contains all true premises (or premise), but a false conclusion, then the argument is invalid.

If the table contains no row showing true premise(s) and a false conclusion, the argument is valid.



P

Q

P

$\supset$

Q

P

/

Q







P	Q	P	$\supset$	Q		P	/	Q
T	T							
T	F							
F	T							
F	F							



P	Q	P	$\supset$	Q		P	/	Q
T	T	T				T		
T	F	T				T		
F	T	F				F		
F	F	F				F		



P	Q	P	$\supset$	Q		P	/	Q
T	T	T		T		T		T
T	F	T		F		T		F
F	T	F		T		F		T
F	F	F		F		F		F





P	Q	P	$\supset$	Q		P	/	Q
T	T	T		T		T		T
T	F	T		F		T		F
F	T	F		T		F		T
F	F	F		F		F		F





P	Q	P	$\supset$	Q		P	/	Q
T	T	T	T	T		T		T
T	F	T	F	F		T		F
F	T	F	T	T		F		T
F	F	F	T	F		F		F





P	Q	P	$\cup$	Q		P	/	Q
T	T	T	T	T		T		T
T	F	T	F	F		T		F
F	T	F	T	T		F		T
F	F	F	T	F		F		F



## Rule for Validity Test:

If there is any row on the truth-table that contains all true premises (or premise), but a false conclusion, then the argument is invalid.

If the table contains no row showing true premise(s) and a false conclusion, the argument is valid.



P	Q	P	$\cup$	Q		P	/	Q
T	T	T	T	T		T		T
T	F	T	F	F		T		F
F	T	F	T	T		F		T
F	F	F	T	F		F		F





P	Q	P	$\cup$	Q		P	/	Q
T	T	T	T	T		T		T
T	F	T	F	F		T		F
F	T	F	T	T		F		T
F	F	F	T	F		F		F





P	Q	P	$\cup$	Q		P	/	Q
T	T	T	T	T		T		T
T	F	T	F	F		T		F
F	T	F	T	T		F		T
F	F	F	T	F		F		F



*It's Valid!*







NOT  
Sorry

The word "NOT" is written in a large, dark, semi-transparent, sans-serif font at the top. Below it, the word "Sorry" is written in a bright orange-red, glowing neon script font.

Use truth-table  
analysis to assess the  
following for validity...



1.  $P \supset Q; \sim P; \therefore \sim Q$

2.  $P \supset Q; Q; \therefore P$

3.  $\sim(P \& Q); P; \therefore \sim Q$

4.  $P \supset Q; P; \therefore Q$

5.  $P \vee Q; \sim P; \therefore Q$

6.  $P \supset Q; \sim Q; \therefore \sim P$

—