Quantifier Rules

Questions yet to be answered:

- [ ] Aristotle or the Stoics?
- [ ] Aristotle or Boole?
- [ ] Is Logicism true?
- [x] How do we know Stoic argument forms are actually valid?

"The logic of propositions, which [the Stoics] studied, is more fundamental than the logic of general terms, which Aristotle studied…

Aristotle's syllogistic takes its place as a fragment of general logic in which theorems of primary logic are assumed without explicit formulation, while the dialectic of Chrysippus appears as the first version of primary logic" (Kneale and Kneale, 1984, 175-76).

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Quantifiers:

$(\forall x)$

$(\exists x)$

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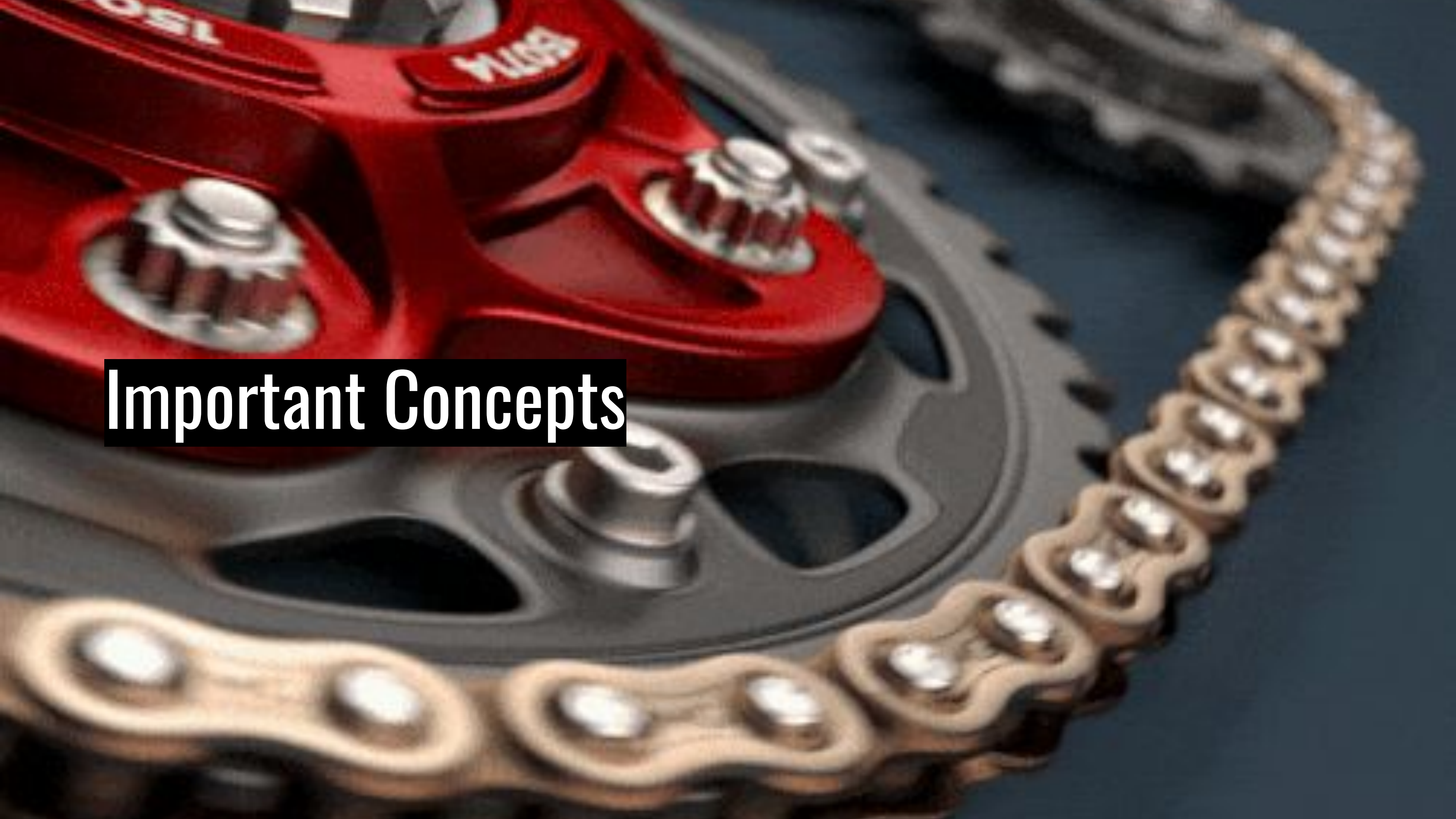
Important Concepts



PD

PD will be the name of our derivation system for making proofs with PL.

Quantifier Rules (the easy ones)

Universal Elimination

$$(\forall x)P$$

$$P(a/x)$$

This means you replace the variable (w-z) with a singular term (a-v).

For example... All philosophers are somewhat strange. Socrates is a philosopher. So, Socrates is somewhat strange.

Derive: Ss

1.	$(\forall y)(Py \supset Sy)$	(As.)
2.	Ps	(As.)
3.		
4.		

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Derive: Ss

1.	$(\forall y)(Py \supset Sy)$	(As.)
2.	Ps	(As.)
3.	$Ps \supset Ss$	1 $\forall$E
4.		

For example… All philosophers are somewhat strange. Socrates is a philosopher. So, Socrates is somewhat strange.

Derive: Ss

1.	$(\forall y)(Py \supset Sy)$	(As.)
2.	Ps	(As.)
3.	$Ps \supset Ss$	$1 \forall E$
4.	Ss	$2, 3 \supset E$

Existential Introduction

$$\begin{array}{|l} P(a/x) \\ \\ (\exists x)P \end{array}$$

You basically just replace the singular term (a-v) with a quantifier and a matching variable (w-z).

For example… Jed knows PL. So, *someone* knows PL.

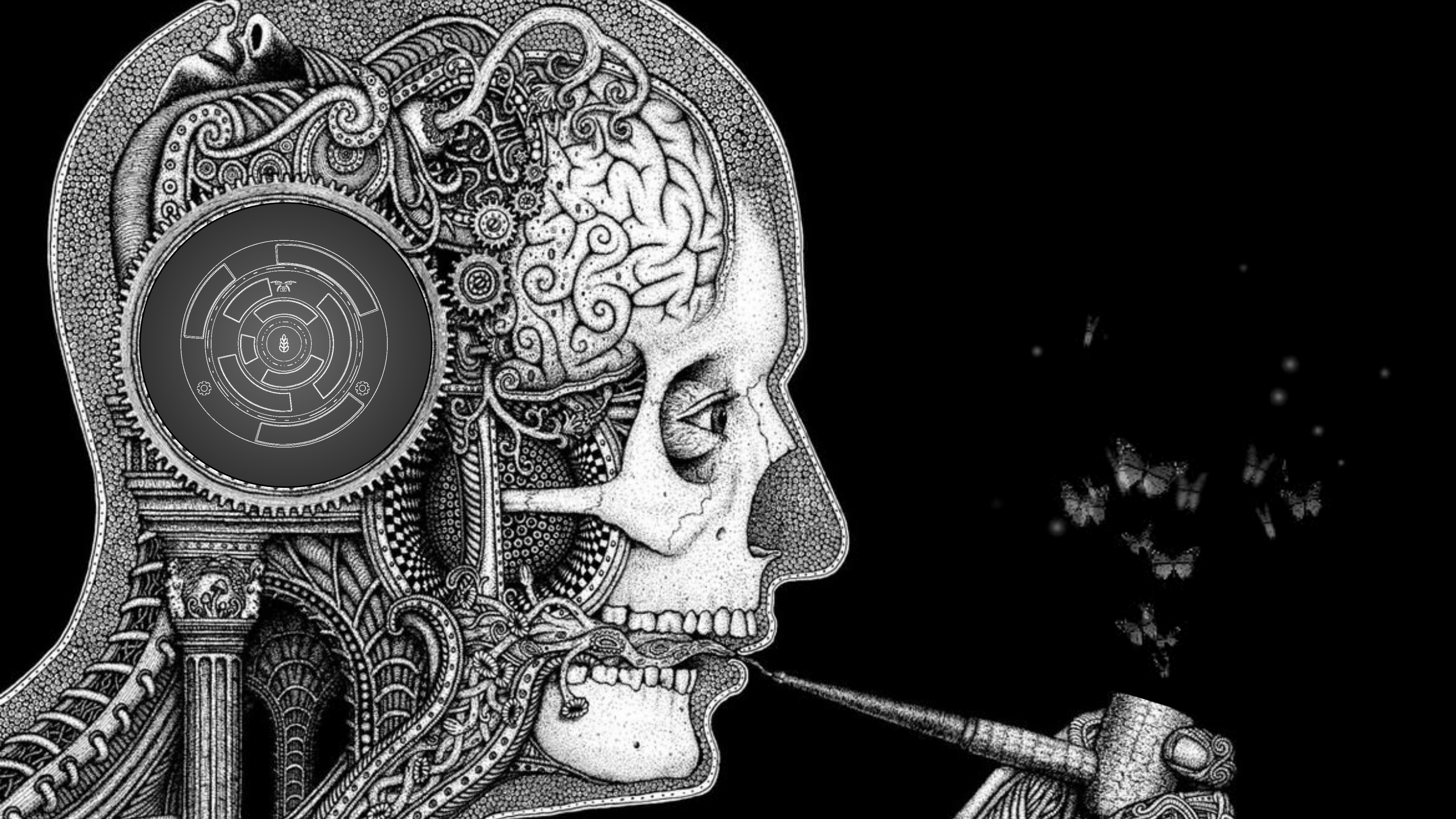
Derive: $(\exists x)(Kxp)$

1.	Kjp	(As.)
2.		

For example... Jed knows PL. So, *someone* knows PL.

Derive: $(\exists x)(Kxp)$

1.	Kjp	(As.)
2.	$(\exists x)(Kxp)$	1 ∃I



Quantifier Rules (the hard ones)

Universal Introduction

Provided that:

1. **a** does not occur in an open assumption (the premises), and
2. **a** does not occur in (∀x)**P** (the target sentence).

| **P(a/x)**
|
| (∀x)**P**

For example...

Derive: (∀y)Fy

1.	Fb & ~ Fc	(As.)
2.	Fb	1 &E
3.	(∀y)Fy	2 ∀I

MISTAKE

Universal Introduction

Provided that:

1. **a** does not occur in an open assumption (the premises), and
2. **a** does not occur in (∀x)**P** (the target sentence).

$$
\begin{array}{|l}
\mathbf{P(a/x)} \\
\\
(\forall \mathbf{x})\mathbf{P}
\end{array}
$$

For example...

Derive: $(\forall y)Fy$

1.	$Fb \;\&\; \sim Fc$	(As.)
2.	Fb	1 &E
3.	$(\forall y)Fy$	2 $\forall$I

MISTAKE

For example...

Derive: $(\forall y)Fy$

1.	$(\forall x)Fx$	(As.)
2.	Fb	1 ∀E
3.	$(\forall y)Fy$	2 ∀I

TOTALLY COOL

Existential Elimination

Provided that:

1. **a** does not occur in an open assumption (the premises).
2. **a** does not occur in (∃x)**P**.
3. **a** does not occur in **Q**.

```
| (∃x)P
|
|   | P(a/x)
|   |----------
|   | Q
|
| Q
```

For example...

Derive: $(\exists x)(Gx \lor Fx)$

1.	$(\exists z)Fz \& (\forall y)Hy$	(As.)
2.	$(\exists z)Fz$	1 &E
3.	$\quad Fb$	As/ $\exists$E
4.	$\quad Gb \lor Fb$	3 $\lor$I
5.	$\quad (\exists x)(Gx \lor Fx)$	4 $\exists$I
6.		2, 3, 5

1	$(\forall x)Fx$	As.
2	Fa	$1\ \forall E$
3	$(\forall y)Fy$	$2\ \forall I$

1	Fb	As.
2	Gb	As.
3	$Fb \& Gb$	1, 2 &I
4	$(\exists x)(Fx \& Gx)$	3 $\exists$I

1	$(\forall x)(\forall y)Hxy$	As.
2	$(\forall y)Hay$	1 $\forall$E
3	Hab	2 $\forall$E
4	$(\exists y)Hay$	3 $\exists$I
5	$(\exists x)(\exists y)Hxy$	4 $\exists$I

1	$(\exists x)(Fx \& Gx)$	As.
2	$Fa \& Ga$	As/$\exists$E
3	Fa	2 &E
4	$(\exists y)Fy$	3 $\exists$I
5	Ga	2 &E
6	$(\exists w)Gw$	5 $\exists$I
7	$(\exists y)Fy \& (\exists w)Gw$	4,6 &I
8	$(\exists y)Fy \& (\exists w)Gw$	1, 2-7 $\exists$E

Read p. 474-490.
Do 10.1**E** #2, p. 491
(If you’re brave:
Try 10.1**E** #1, p. 490)