

Replacement Rules (Pt. I)

A close-up photograph of a motorcycle's chain drive system. A bright red metal component, likely a chain guide or tensioner, is positioned at the top left. It features several silver-colored bolts and a small gear-like structure. Below it, a large black metal gear with multiple teeth is visible. A gold-colored chain with silver-colored pins is wrapped around the gear, extending towards the right side of the frame. The background is a dark, out-of-focus blue-grey color.

Replacement Rules: Important Concepts

Fundamental Concepts in SD:

Non-subderivational Rules

- $\&I$
- $\&E$
- $\supset E$
- $\equiv E$
- $\forall I$

Subderivational Rules

- $\supset I$
- $\sim E$
- $\sim I$
- $\equiv I$
- $\forall E$

SD+

SD+ is a derivation system that includes all the derivation rules from SD, but also **replacement rules** (which can radically shorten the length of a proof).

Derivable in SD^+

A sentence \mathbf{P} of TL is derivable in SD^+ from a set Γ of sentences of TL if and only if there is a derivation in SD^+ in which all the primary assumptions are members of Γ and \mathbf{P} occurs within the scope of only those assumptions.

Most Important Replacement Rules

De Morgan's Rule (DM)

Where **P** and **Q** are variables ranging over declarative sentences...

Anywhere in a proof, $\sim(\mathbf{P} \ \& \ \mathbf{Q})$ may replace or be replaced by

$\sim\mathbf{P} \vee \sim\mathbf{Q}$;

$\sim(\mathbf{P} \vee \mathbf{Q})$ may replace or be replaced by

$\sim\mathbf{P} \ \& \ \sim\mathbf{Q}$.

\sim	(P	&	Q)		\sim	P	v	\sim	Q
F	T	T	T		F	T	F	F	T
T	T	F	F		F	T	T	T	F
T	F	F	T		T	F	T	F	T
T	F	F	F		T	F	T	T	F

\sim	(P	&	Q)		\sim	P	v	\sim	Q
F	T	T	T		F	T	F	F	T
T	T	F	F		F	T	T	T	F
T	F	F	T		T	F	T	F	T
T	F	F	F		T	F	T	T	F

Modus Tollens (MT)

Where **P** and **Q** are variables ranging over declarative sentences...

1. Given: $P \supset Q$
2. Given: $\sim Q$
3. You may infer: $\sim P$

Modus Tollens can take many forms...

$A \supset B; \sim B; \therefore \sim A$

$J \supset L; \sim L; \therefore \sim J$

$\sim I \supset \sim O; \sim \sim O; \therefore \sim \sim I$

$(A \vee B) \supset (C \& D); \sim(C \& D); \therefore \sim(A \vee B)$



Disjunctive Syllogism (DS)

Where **P** and **Q** are variables ranging over declarative sentences...

1. Given: **P** v **Q**
2. Given: \sim **P** or (\sim **Q**)
3. You may infer: **Q** (or **P**)

Again...

Disjunctive Syllogisms can take many forms,
and the order the premises are listed in does not matter.

Hypothetical Syllogism (HS)

Where **P** and **Q** are
variables ranging over
declarative sentences...

1. Given: $P \supset Q$
2. Given: $Q \supset R$
3. Inference: $P \supset R$

Again...

Hypothetical Syllogisms can take many forms,
and the order the premises are listed in does not matter.

Implication (Imp)

Where **P** and **Q** are variables ranging over declarative sentences...

Anywhere in a proof, $P \supset Q$ may replace or be replaced by $\sim P \vee Q$.

Double Negation (DN)

Where **P** is a variable
ranging over declarative
sentences...

Anywhere in a proof,
P may replace or be
replaced by $\sim\sim\mathbf{P}$ (or
replace $\sim\sim\mathbf{P}$ with **P**).



This!

NOT
Sorry