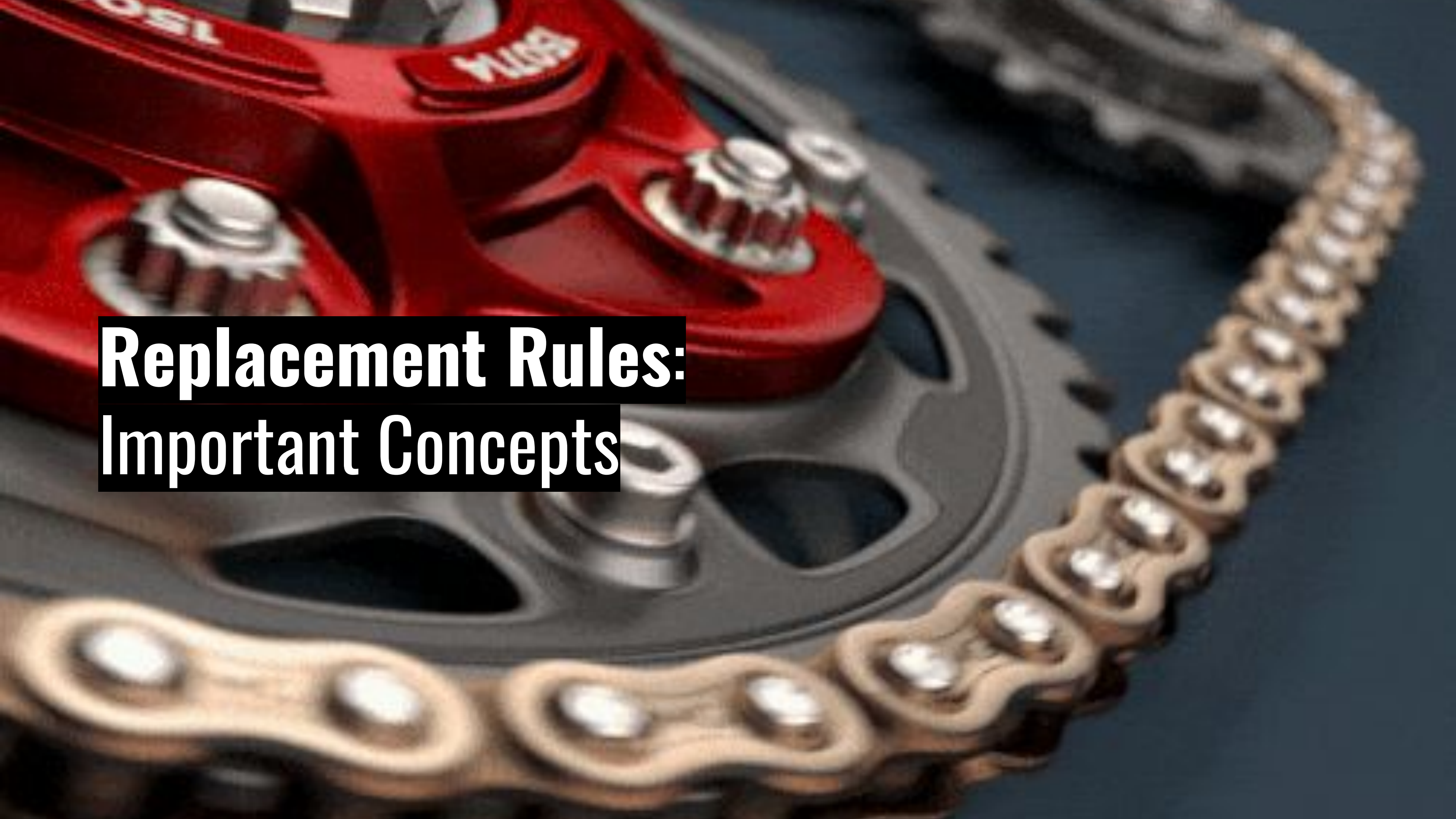
Replacement Rules (Pt. I)



Replacement Rules:
Important Concepts

Fundamental Concepts in SD:

Non-subderivational Rules

- &I
- &E
- ⊃E
- ≡E
- ∨I

Subderivational Rules

- ⊃I
- ~E
- ~I
- ≡I
- ∨E

SD+

SD+ is a derivation system that includes all the derivation rules from SD, but also **replacement rules** (which can radically shorten the length of a proof).

Derivable in SD+

A sentence **P** of TL is derivable in *SD+* from a set Γ of sentences of TL if and only if there is a derivation in *SD+* in which all the primary assumptions are members of Γ and **P** occurs within the scope of only those assumptions.

Most Important Replacement Rules

De Morgan's Rule (DM)

Where **P** and **Q** are variables ranging over declarative sentences…

Anywhere in a proof, ~(**P** & **Q**) may replace or be replaced by ~**P** v ~**Q**; ~(**P** v **Q**) may replace or be replaced by ~**P** & ~**Q**.

~	(P	&	Q)		~	P	v	~	Q
F	T	T	T		F	T	F	F	T
T	T	F	F		F	T	T	T	F
T	F	F	T		T	F	T	F	T
T	F	F	F		T	F	T	T	F

~	(P	&	Q)		~	P	v	~	Q
F	T	T	T		F	T	F	F	T
T	T	F	F		F	T	T	T	F
T	F	F	T		T	F	T	F	T
T	F	F	F		T	F	T	T	F

Modus Tollens (MT)

Where **P** and **Q** are variables ranging over declarative sentences...

1. Given: **P** ⊃ **Q**
2. Given: ~**Q**
3. You may infer: ~**P**

Modus Tollens can take many forms…

A ⊃ B; ~B; ∴ ~A

J ⊃ L; ~L; ∴ ~J

~I ⊃ ~O; ~~O; ∴ ~~I

(A v B) ⊃ (C & D); ~(C & D); ∴ ~(A v B)

Disjunctive Syllogism (DS)

Where **P** and **Q** are variables ranging over declarative sentences…

1. Given: **P** v **Q**
2. Given: ~**P** or (~**Q**)
3. You may infer: **Q** (or **P**)

Again…

Disjunctive Syllogisms can take many forms,
and the order the premises are listed in does not matter.

Hypothetical Syllogism (HS)

Where **P** and **Q** are variables ranging over declarative sentences…

1. Given: **P** ⊃ **Q**
2. Given: **Q** ⊃ **R**
3. Inference: **P** ⊃ **R**

Again...

Hypothetical Syllogisms can take many forms,

and the order the premises are listed in does not matter.

Implication (Imp)

Where **P** and **Q** are variables ranging over declarative sentences…

Anywhere in a proof, **P ⊃ Q** may replace or be replaced by **~P ∨ Q**.

Double Negation (DN)

Where **P** is a variable ranging over declarative sentences…

Anywhere in a proof, **P** may replace or be replaced by **~~P** (or replace **~~P** with **P**).


DO STUFF

This!