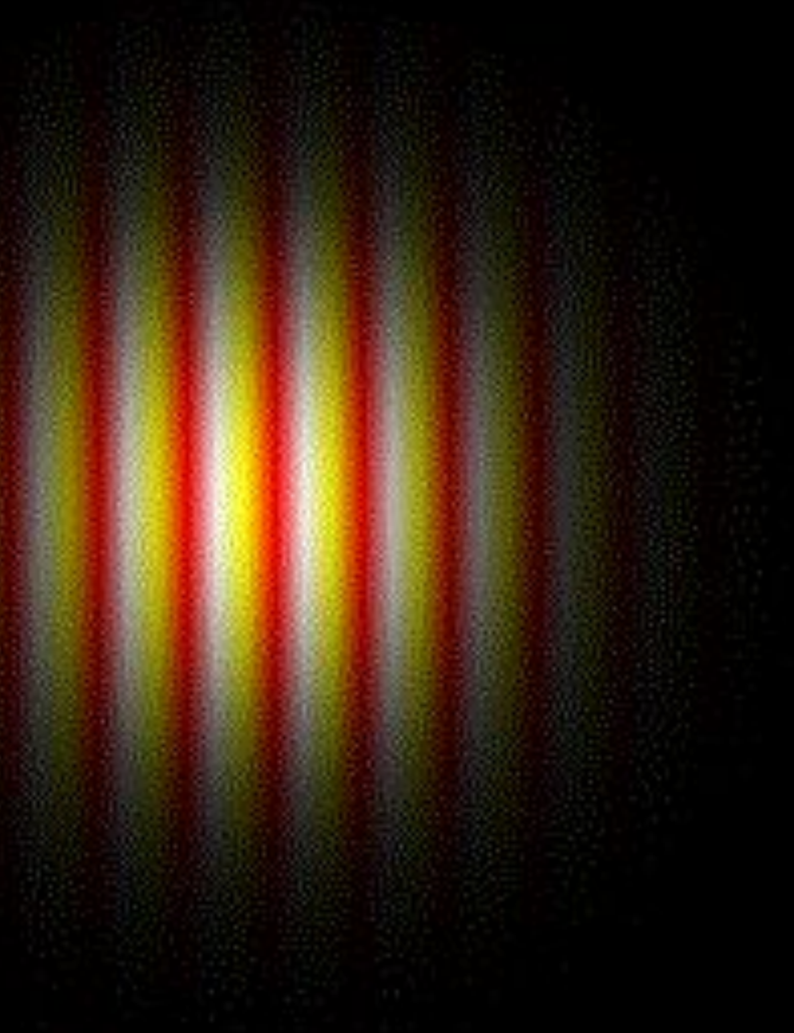
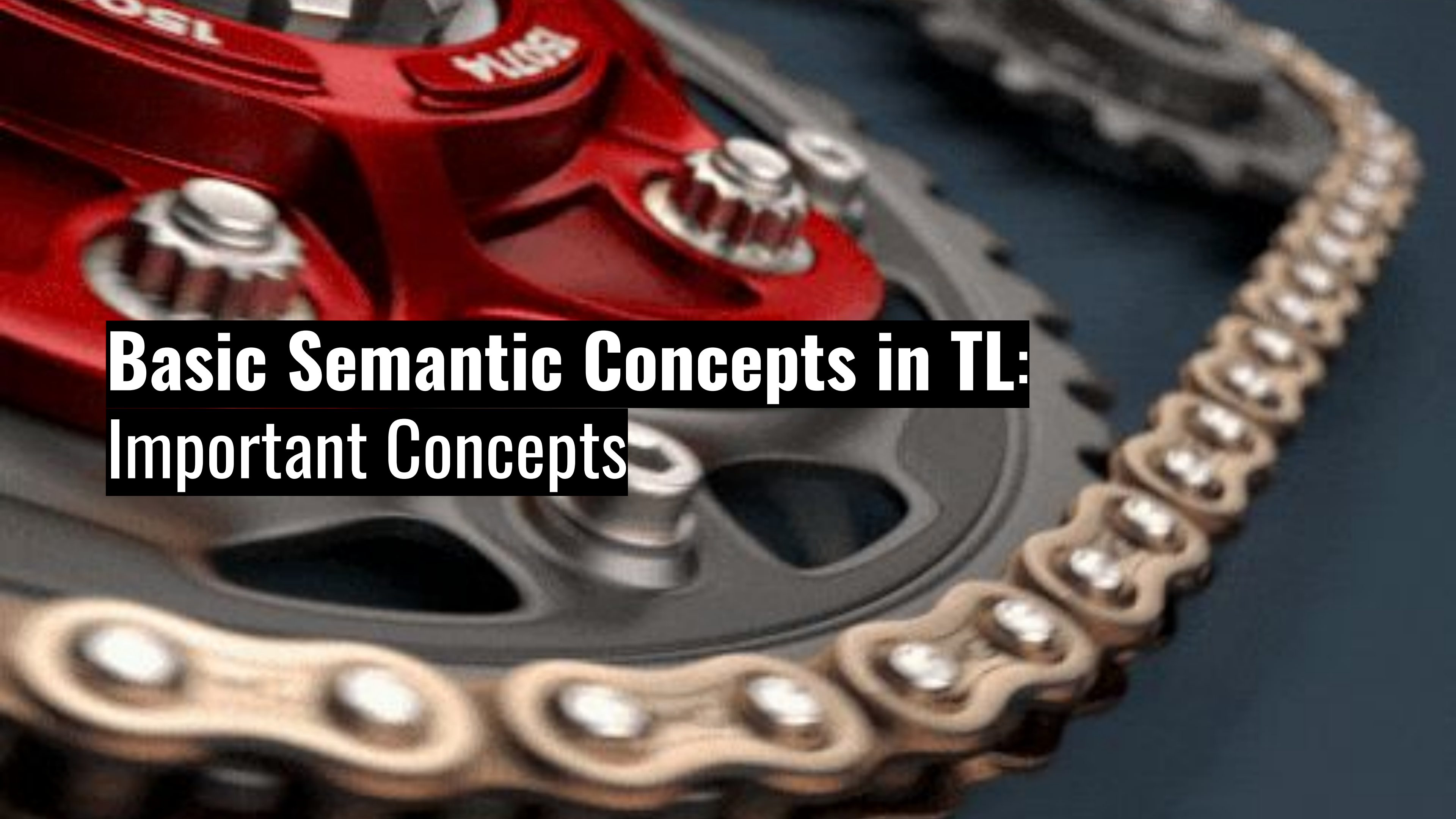


Truth-Tables (Pt. II)



Basic Semantic Concepts in TL:

Important Concepts

Set theory is the mathematical theory of well-determined collections, called *sets*, of objects that are called *members*, or *elements*, of the set.

Sets are denoted by curly braces, i.e., “{ }”.

Basics

- {A, B ⊃ (C ⊃ B), D ∨ [~ (C & A)]} – a <u>set</u> of three sentences of TL.
- 'A', 'B ⊃ (C ⊃ B)', and 'D ∨ [~ (C & A)]' are <u>members</u> of {A, B ⊃ (C ⊃ B), D ∨ [~ (C & A)]}.
- ∅ is (the name of) the empty set.

- The variable 'Γ' (Greek gamma) is used to talk generally about sets of sentences of TL.
- {P} is the unit set of P.
- The union of two sets Γ_1 and Γ_2, $\Gamma_1 \cup \Gamma_2$, is a set containing all and only members of Γ_1 and Γ_2.

A truth-value assignment is an assignment of truth-values to the simple (or atomic) sentences of TL (*all of them*, strictly speaking).

A	⊃	B
T		T
T		F
F		T
F		F

A truth-value assignment is an assignment of truth-values to the simple (or atomic) sentences of TL (*all of them*, strictly speaking).

A	⊃	B
T	T	T
T	F	F
F	T	T
F	T	F

Logical truth: A sentence is logically true just in case it is not possible for it to be false.

↓

Truth-functional truth: A sentence of TL is truth-functionally true if and only if it is true on every truth-value assignment.

Logical falsity: A sentence is logically false just in case it is not possible for it to be true.

↓

Truth-functional falsity: A sentence of TL is truth-functionally false if and only if it is false on every truth-value assignment.

Logical indeterminacy: A sentence is logically indeterminate just in case it is neither logically true nor logically false.

↓

Truth-functional indeterminacy: A sentence of TL is truth-functionally indeterminate if and only if it is neither truth-functionally true nor truth-functionally false.

Question:
What does it take to demonstrate that a sentence is truth-functionally false?

B	D	~	(D	⊃	((B	∨	D)	⊃	D))
T	T	F	T	T	T	T	T	T	T
T	F	F	F	T	T	T	F	F	F
F	T	F	T	T	F	T	T	T	T
F	F	F	F	T	F	F	F	T	F

Question:

What does it take to demonstrate that a sentence is truth-functionally true?

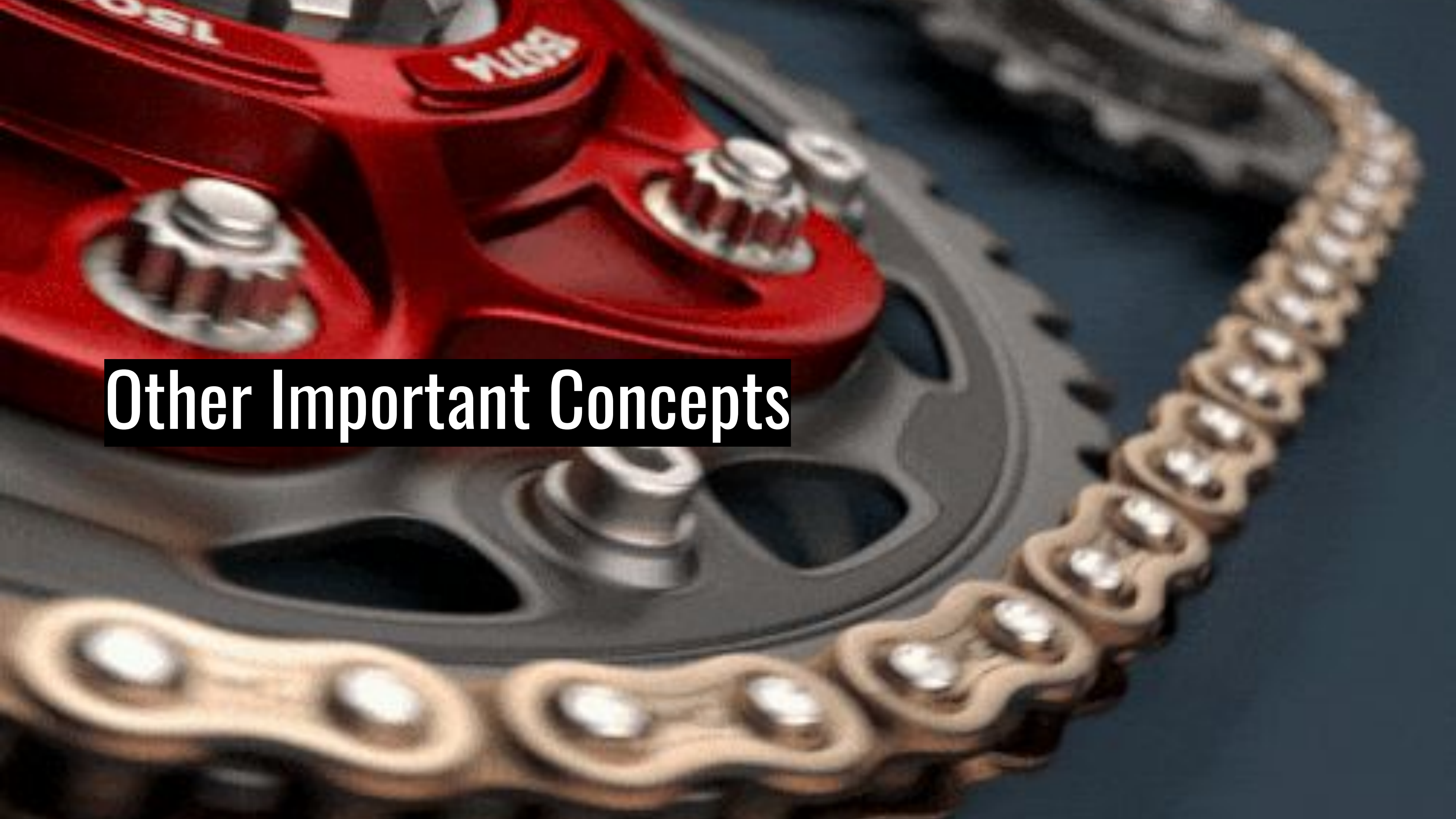
B	D	D	⊃	((B	∨	D)	⊃	D)
T	T	T	T	T	T	T	T	T
T	F	F	T	T	T	F	F	F
F	T	T	T	F	T	T	T	T
F	F	F	T	F	F	F	T	F

Question:

What does it take to demonstrate that a sentence is truth-functionally indeterminate?

B	D	(B	∨	D)	⊃	D
T	T	T	T	T	T	T
T	F	T	T	F	F	F
F	T	F	T	T	T	T
F	F	F	F	F	T	F

Other Important Concepts



One statement implies a second statement if whenever the first statement is true the second one *must* be true.

Two sentences are equivalent just in case they imply each other; i.e., equivalence is a logical relation that holds between two statements P and Q when P implies Q ***and*** Q implies P.

Two or more sentences are logically consistent if it is possible that they are all true simultaneously.

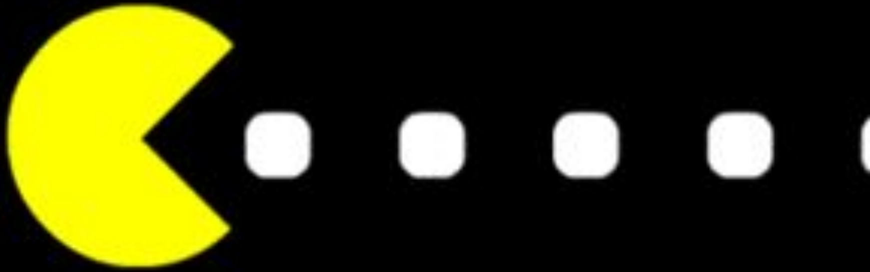
Consider the following statements:
A: Rodrigo weighs exactly 200lbs.
B: Rodrigo weighs more than 100lbs.

Here A *implies* B.

Question:
Are these statements equivalent?

Are the following statements equivalent?

A: It is not the case that either Aristo or Blipo is home.
B: Aristo is not home and Blipo is not home.



Food for thought…

Question:
How can this help resolve disagreement?

Reasoning can now be broken down into its basic elements. Disagreements, in theory, can be resolved with precision.

How?

Step 1: Translate the argument from English into TL.

Step 2. Apply the exact methods of Logic to assess the validity of the argument.

> Note: Use an empirical discipline to verify the truth of the premises.

Step 3: Derive precise answers.

Are the following statements equivalent?

A: ~(A v B)

B: ~A & ~B

Then....

Truth-table Analysis

...

EQUIVALENCE/CONSISTENCY

EDITION

Steps

#1: Follow all steps from ARGUMENT EDITION.

Rule for Equivalence Test:

If two formulas on top of a table have matching final columns, then they are equivalent. If the final columns do not match, then the formulas are not equivalent.

Are the following statements equivalent?

A: ~(A v B)

B: ~A & ~B

Then….

A B ~ (A ∨ B) ~ A & ~ B

A	B	~	(A	∨	B)		~	A	&	~	B

A	B	~	(A	∨	B)		~	A	&	~	B
T	T		T		T			T			T
T	F		T		F			T			F
F	T		F		T			F			T
F	F		F		F			F			F

A	B	~	(A	∨	B)		~	A	&	~	B
T	T		T		T			T			T
T	F		T		F			T			F
F	T		F		T			F			T
F	F		F		F			F			F

A	B	~	(A	∨	B)		~	A	&	~	B
T	T		T	T	T		F	T		F	T
T	F		T	T	F		F	T		T	F
F	T		F	T	T		T	F		F	T
F	F		F	F	F		T	F		T	F

A	B	~	(A	∨	B)		~	A	&	~	B
T	T	F	T	T	T		F	T	F	F	T
T	F	F	T	T	F		F	T	F	T	F
F	T	F	F	T	T		T	F	F	F	T
F	F	T	F	F	F		T	F	T	T	F

Rule for Equivalence Test:

If two formulas on top of a table have matching final columns, then they are equivalent. If the final columns do not match, then the formulas are not equivalent.

A	B	~	(A	∨	B)		~	A	&	~	B
T	T	F	T	T	T		F	T	F	F	T
T	F	F	T	T	F		F	T	F	T	F
F	T	F	F	T	T		T	F	F	F	T
F	F	T	F	F	F		T	F	T	T	F


Equivalence!

Rule for Consistency Test:
Given two (or more) formulas side by side on top of a table, if there is **at least one** row where the main operator for all the formulas is true, then the sentences are consistent. If there is no row on which the main operators are true, the sentences are inconsistent.

A	B	H	A		B	⊃	H		B

A	B	H	A		B	$\supset$	H		B
T	T	T							
T	T	F							
T	F	T							
T	F	F							
F	T	T							
F	T	F							
F	F	T							
F	F	F							

A	B	H	A	[redacted]	B	⊃	H	[redacted]	B
T	T	T	T	[redacted]	T		T	[redacted]	T
T	T	F	T	[redacted]	T		F	[redacted]	T
T	F	T	T	[redacted]	F		T	[redacted]	F
T	F	F	T	[redacted]	F		F	[redacted]	F
F	T	T	F	[redacted]	T		T	[redacted]	T
F	T	F	F	[redacted]	T		F	[redacted]	T
F	F	T	F	[redacted]	F		T	[redacted]	F
F	F	F	F	[redacted]	F		F	[redacted]	F

A	B	H	A		B	⊃	H		B
T	T	T	T		T	T	T		T
T	T	F	T		T	F	F		T
T	F	T	T		F	T	T		F
T	F	F	T		F	T	F		F
F	T	T	F		T	T	T		T
F	T	F	F		T	F	F		T
F	F	T	F		F	T	T		F
F	F	F	F		F	T	F		F

A	B	H	A		B	⊃	H		B
T	T	T	T		T	T	T		T
T	T	F	T		T	F	F		T
T	F	T	T		F	T	T		F
T	F	F	T		F	T	F		F
F	T	T	F		T	T	T		T
F	T	F	F		T	F	F		T
F	F	T	F		F	T	T		F
F	F	F	F		F	T	F		F


Consistent!

Advanced Techniques:
Indirect Truth-Tables

$$A \supset B, \quad B \supset C, \quad C \supset D, \quad D \supset E \therefore E \supset A$$

A	$\supset$	B		B	$\supset$	C		C	$\supset$	D		D	$\supset$	E		E	$\supset$	A

Step 1:

Form the hypothesis that the argument is invalid.

Recall that if an argument is invalid, it is possible that the premises are all true and the conclusion is false.

Thus, if this argument is invalid, then there must be a consistent assignment of truth-values to the sentence constants of the argument that makes the premises all true and the conclusion false.

A	$\supset$	B		B	$\supset$	C		C	$\supset$	D		D	$\supset$	E		E	$\supset$	A

A	⊃	B		B	⊃	C		C	⊃	D		D	⊃	E		E	⊃	A
	T				T				T				T				F	

Step 2:

Test the hypothesis by trying to fill in the rest of the truth-tables, consistently, based on the hypothesis that the argument is invalid.

If it is possible to assign the truth-values in such a way that the premises are true and the conclusion is false, with no contradictory assignment of truth-values, then the hypothesis is verified.

If it is not possible to consistently assign truth-values so as to make the premises all true and the conclusion false, this would show the argument is valid.

A	⊃	B		B	⊃	C		C	⊃	D		D	⊃	E		E	⊃	A
	T				T				T				T				F	

A	⊃	B		B	⊃	C		C	⊃	D		D	⊃	E		E	⊃	A
	T				T				T				T			T	F	F

A	⊃	B		B	⊃	C		C	⊃	D		D	⊃	E		E	⊃	A
F	T				T				T				T	T		T	F	F

A	$\supset$	B		B	$\supset$	C		C	$\supset$	D		D	$\supset$	E		E	$\supset$	A
F	T	T			T				T				T	T		T	F	F

A	⊃	B		B	⊃	C		C	⊃	D		D	⊃	E		E	⊃	A
F	T	T		T	T				T				T	T		T	F	F

A	⊃	B		B	⊃	C		C	⊃	D		D	⊃	E		E	⊃	A
F	T	T		T	T	T			T				T	T		T	F	F

A	⊃	B		B	⊃	C		C	⊃	D		D	⊃	E		E	⊃	A
F	T	T		T	T	T		T	T				T	T		T	F	F

A	⊃	B		B	⊃	C		C	⊃	D		D	⊃	E		E	⊃	A
F	T	T		T	T	T		T	T	T			T	T		T	F	F

A	⊃	B		B	⊃	C		C	⊃	D		D	⊃	E		E	⊃	A
F	T	T		T	T	T		T	T	T		T	T	T		T	F	F

A	⊃	B		B	⊃	C		C	⊃	D		D	⊃	E		E	⊃	A
F	T	T		T	T	T		T	T	T		T	T	T		T	F	F

INVALID

This!