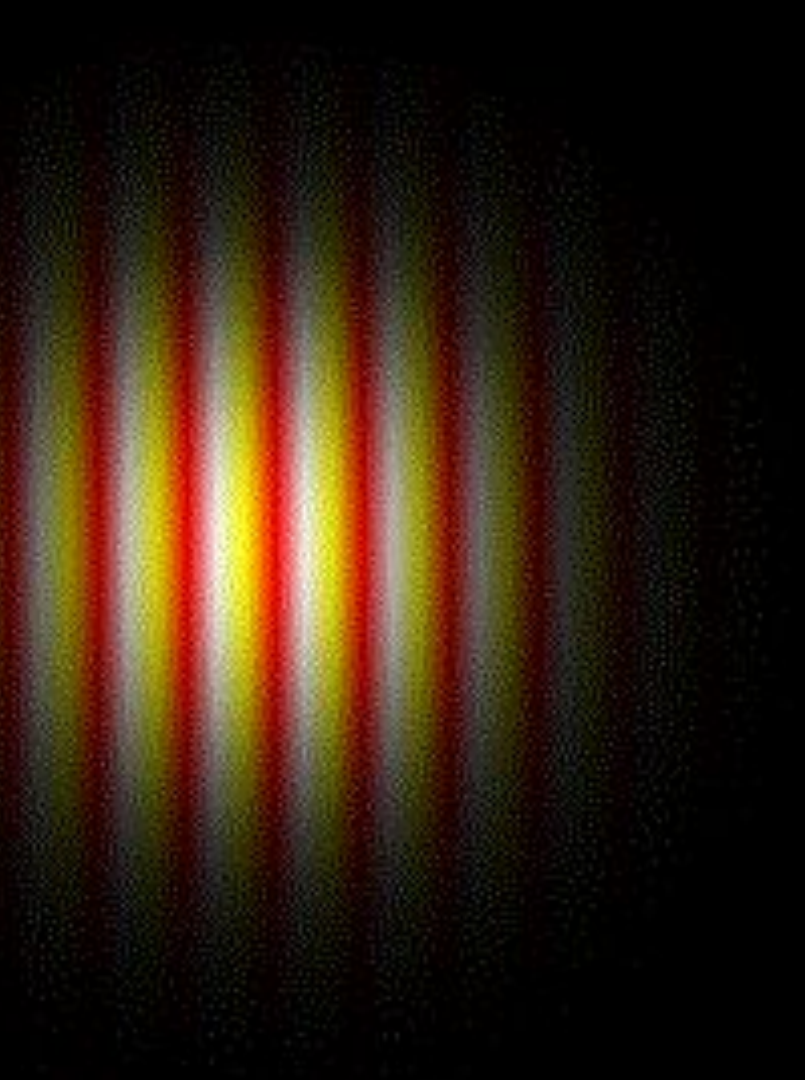


# Conclusions

# Logic and its connection to mathematics...

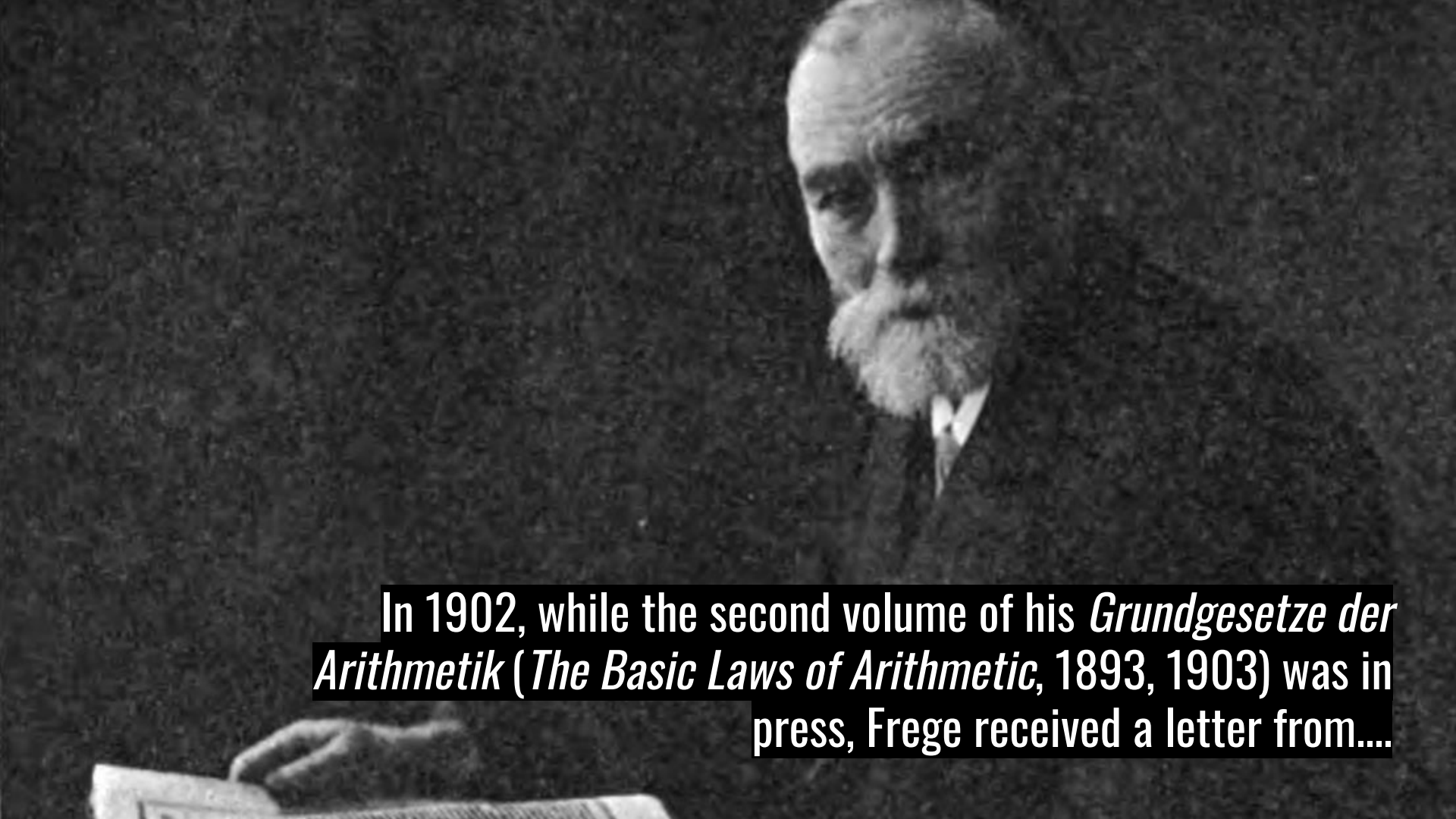
# Questions yet to be answered:

- [x] Aristotle or the Stoics?
- [x] Aristotle or Boole?
- [ ] Is Logicism true?
- [x] How do we know Stoic argument forms are actually valid?

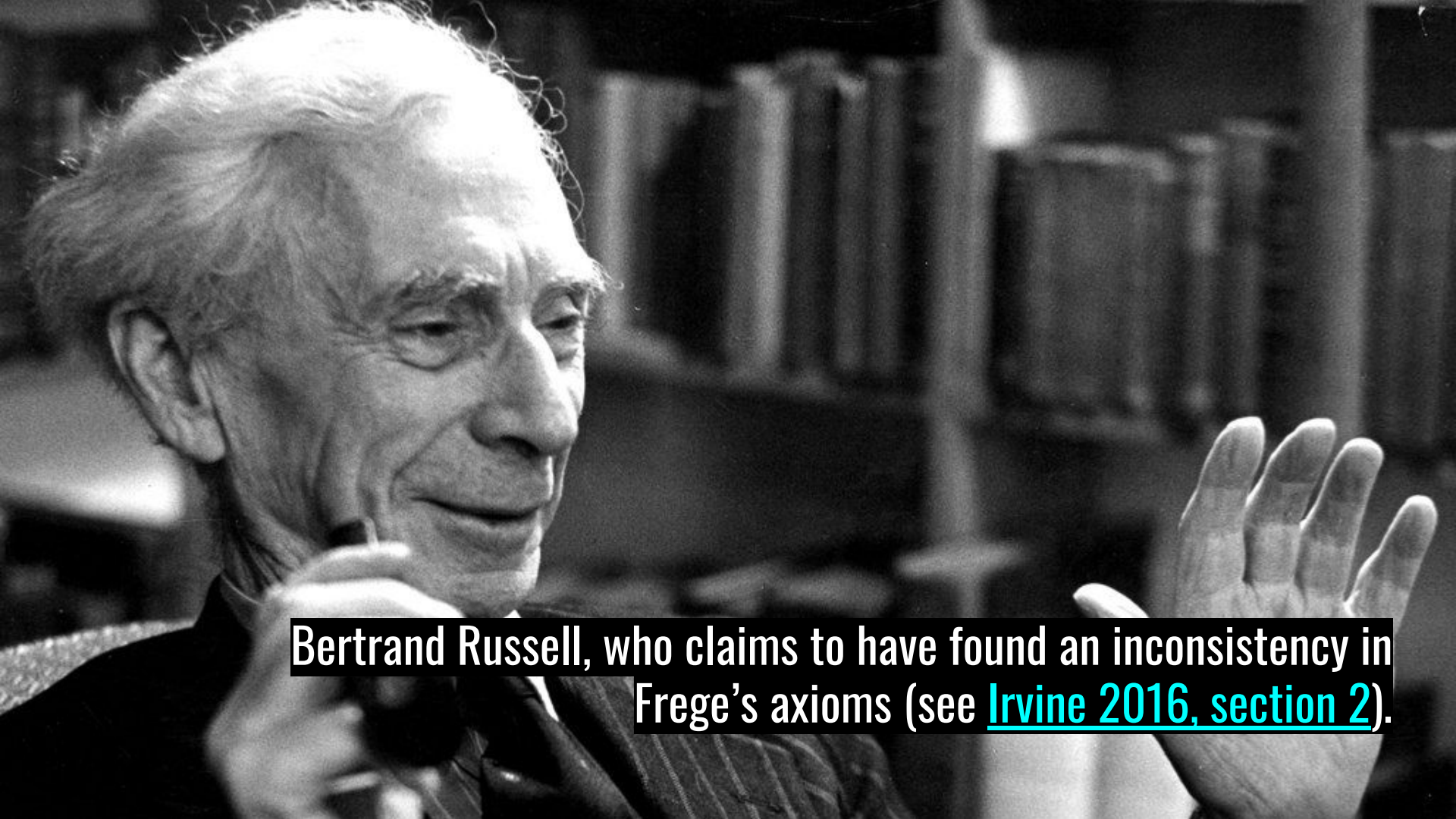
# Is Logicism true?

# Storytime!

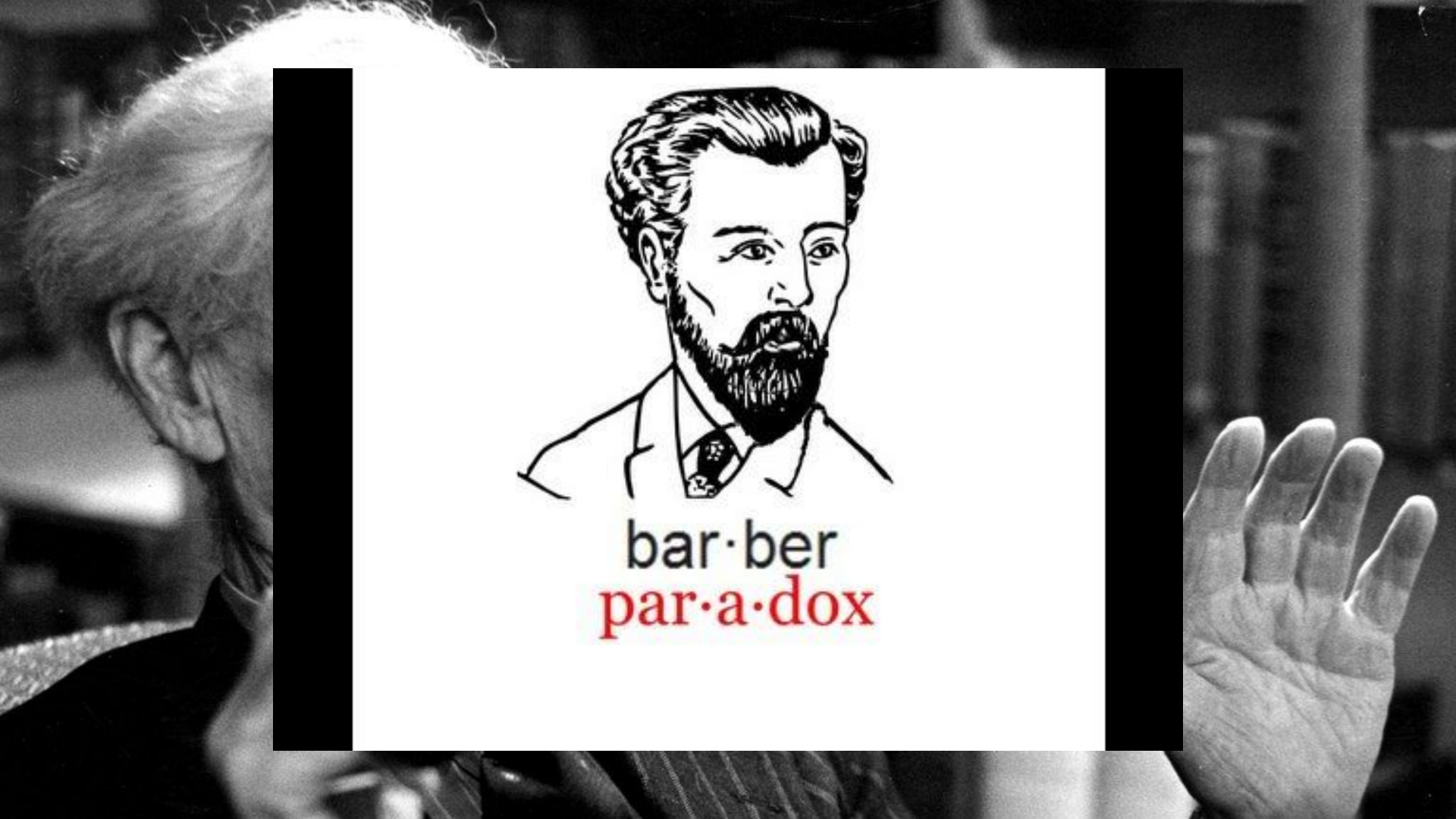
Gottlob Frege publishes *Begriffsschrift*, 1879



In 1902, while the second volume of his *Grundgesetze der Arithmetik* (*The Basic Laws of Arithmetic*, 1893, 1903) was in press, Frege received a letter from....



Bertrand Russell, who claims to have found an inconsistency in Frege's axioms (see Irvine 2016, section 2).


bar·ber
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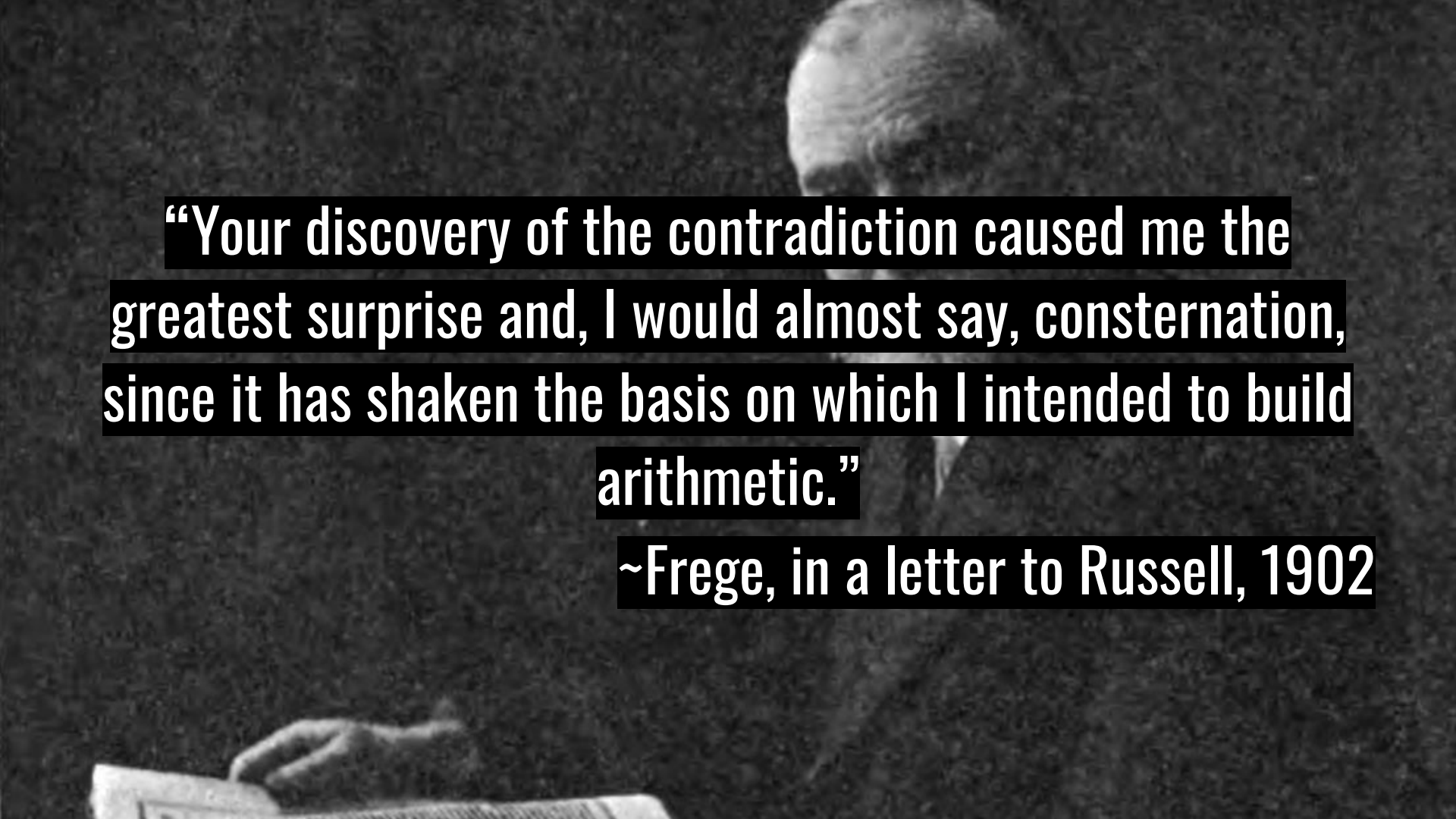
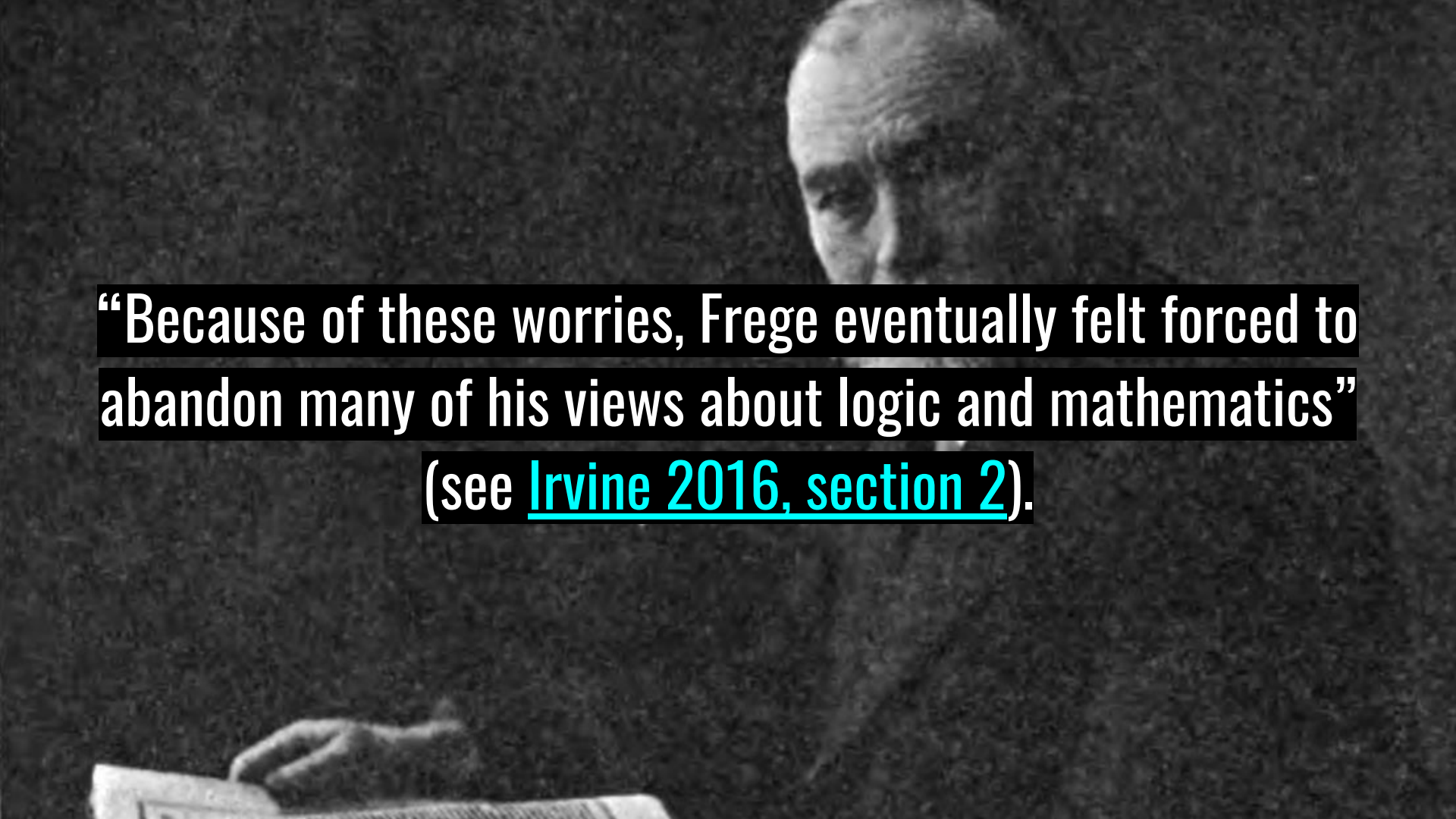
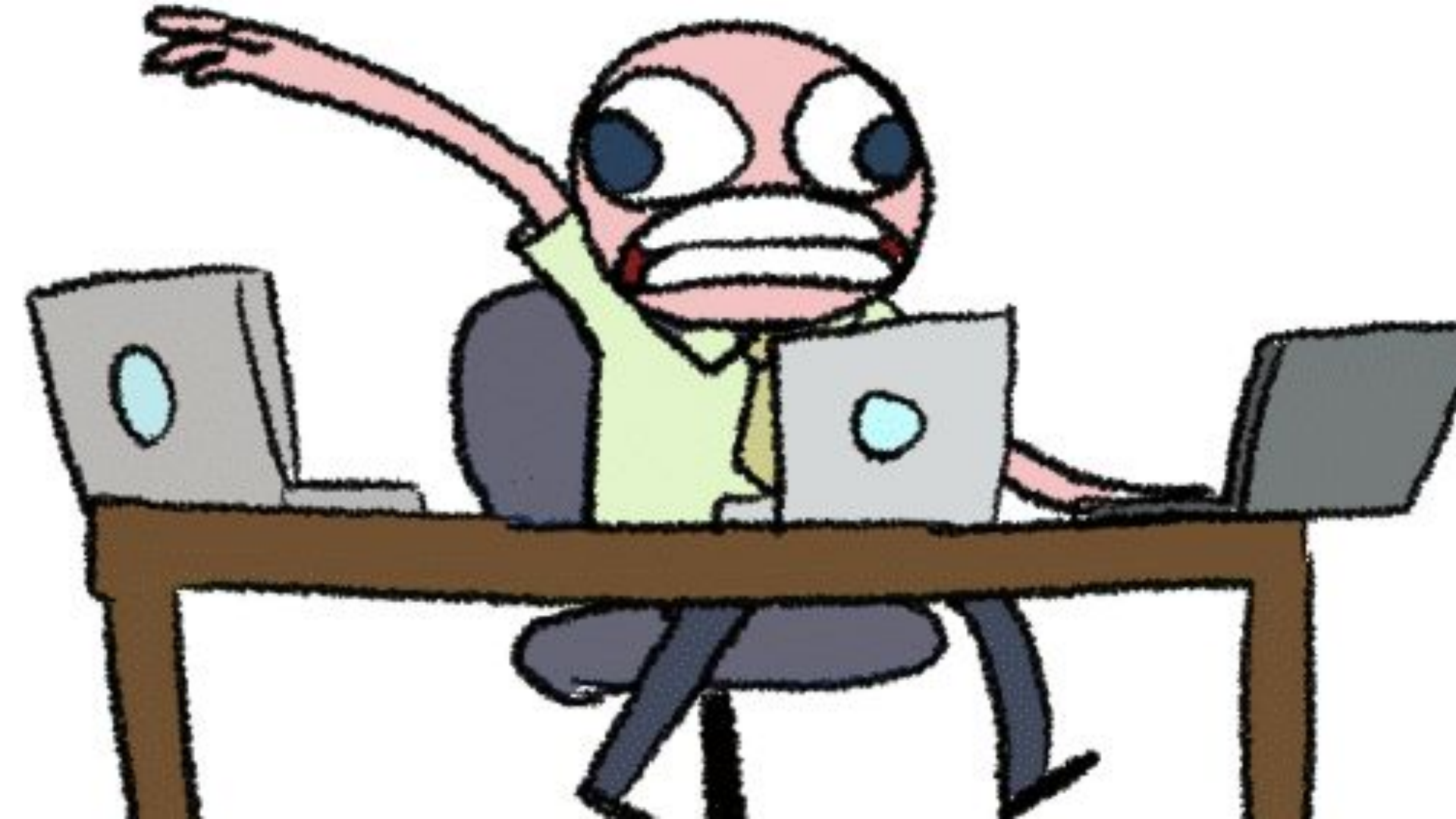
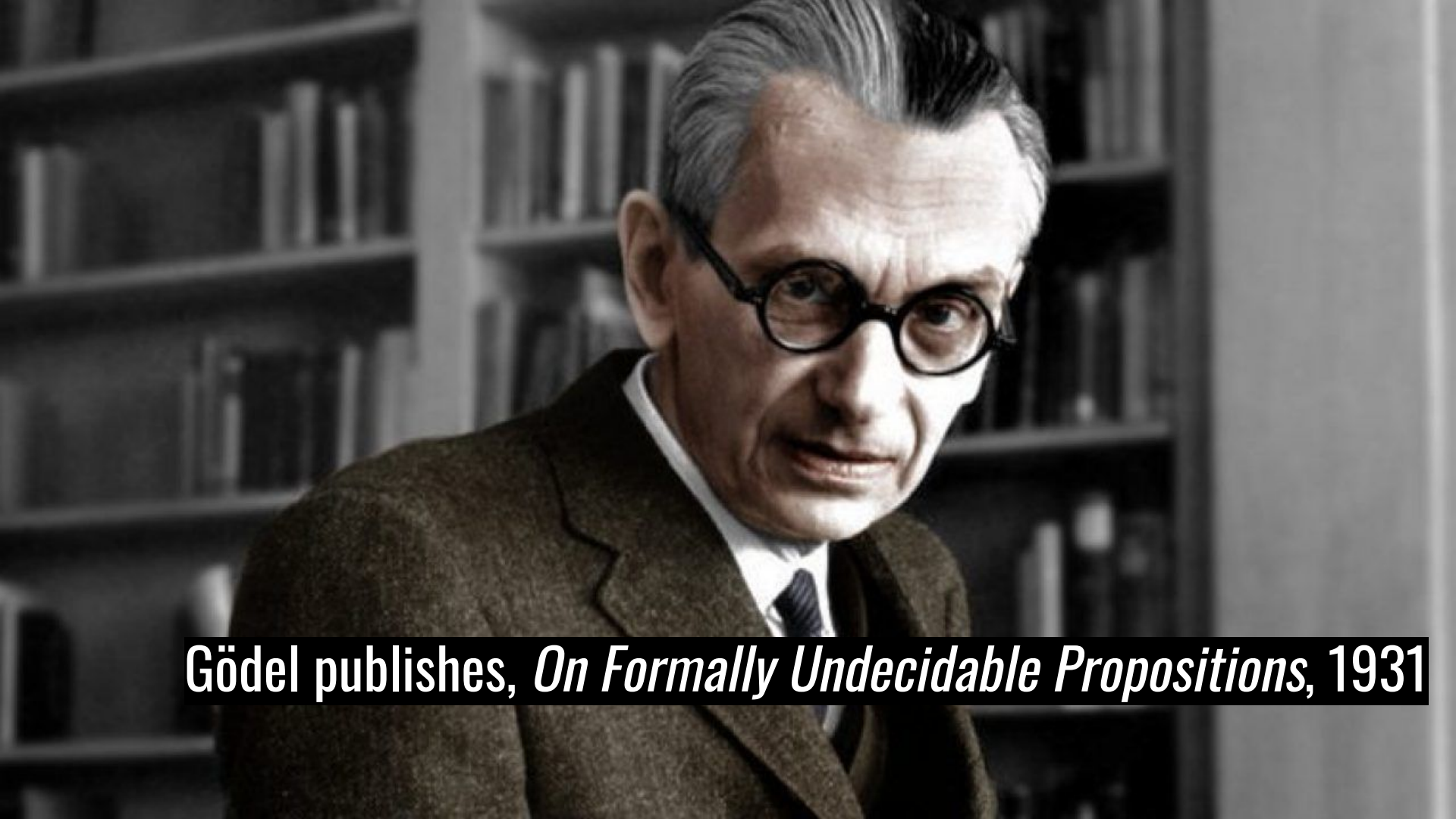

“Your discovery of the contradiction caused me the greatest surprise and, I would almost say, consternation, since it has shaken the basis on which I intended to build arithmetic.”
~Frege, in a letter to Russell, 1902



“Because of these worries, Frege eventually felt forced to abandon many of his views about logic and mathematics” (see Irvine 2016, section 2).



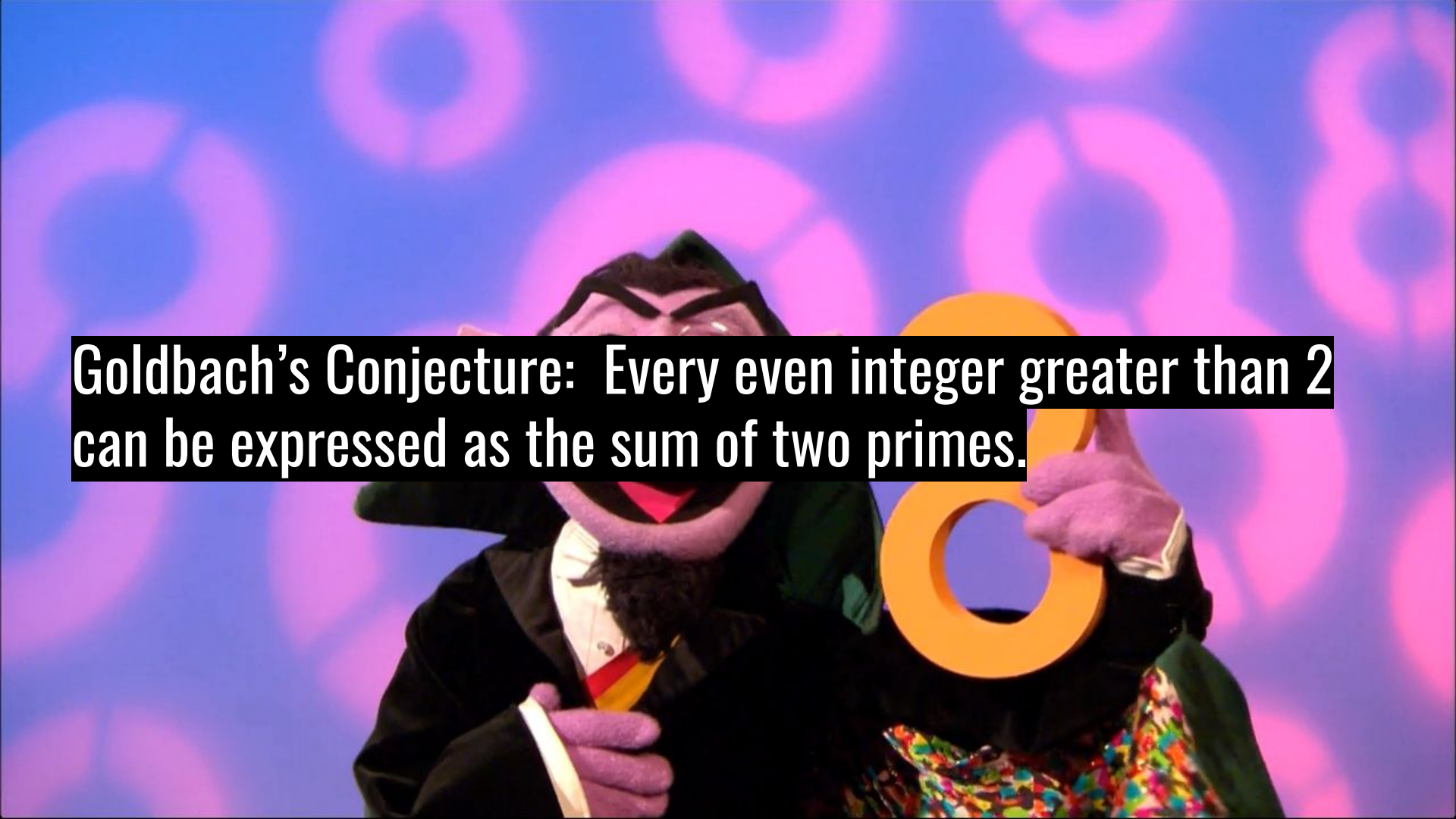


Gödel publishes, *On Formally Undecidable Propositions*, 1931

Gödel's first incompleteness theorem states that:

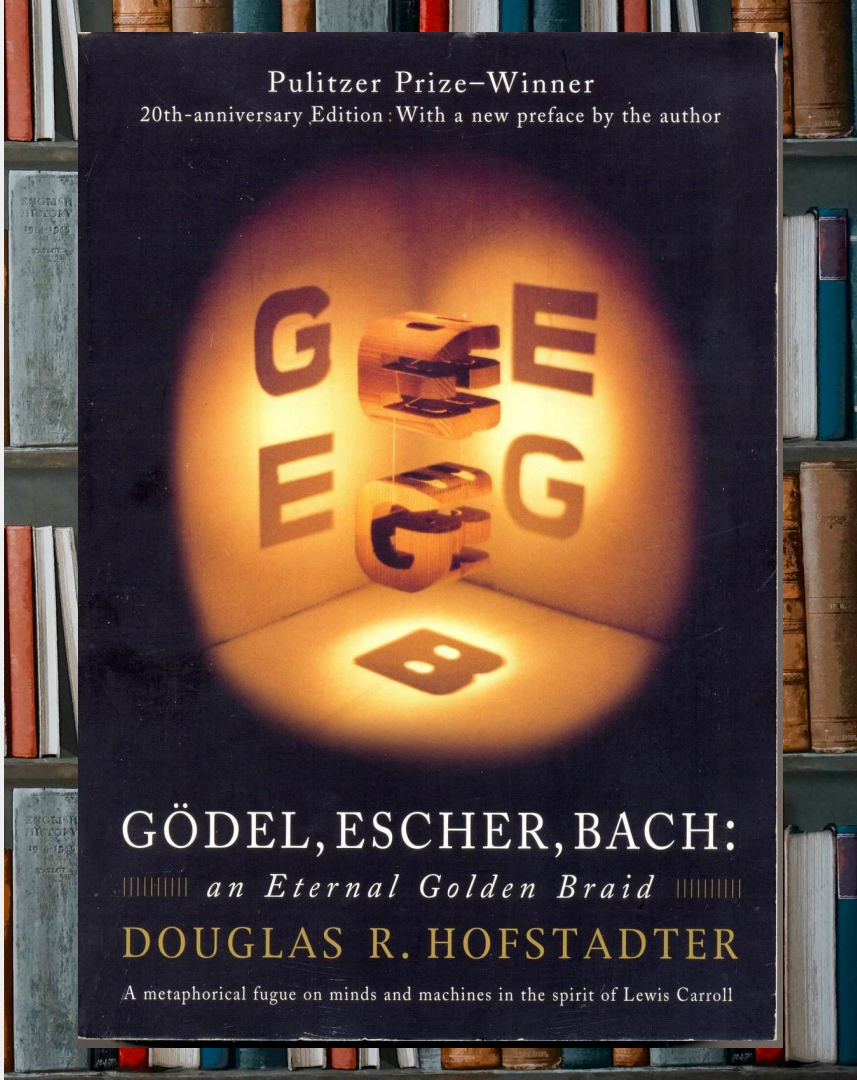
- in any consistent formal system *F* within which a certain amount of arithmetic can be carried out,
- there are statements of the language of *F* which can neither be proved nor disproved in *F*.

Goldbach’s Conjecture

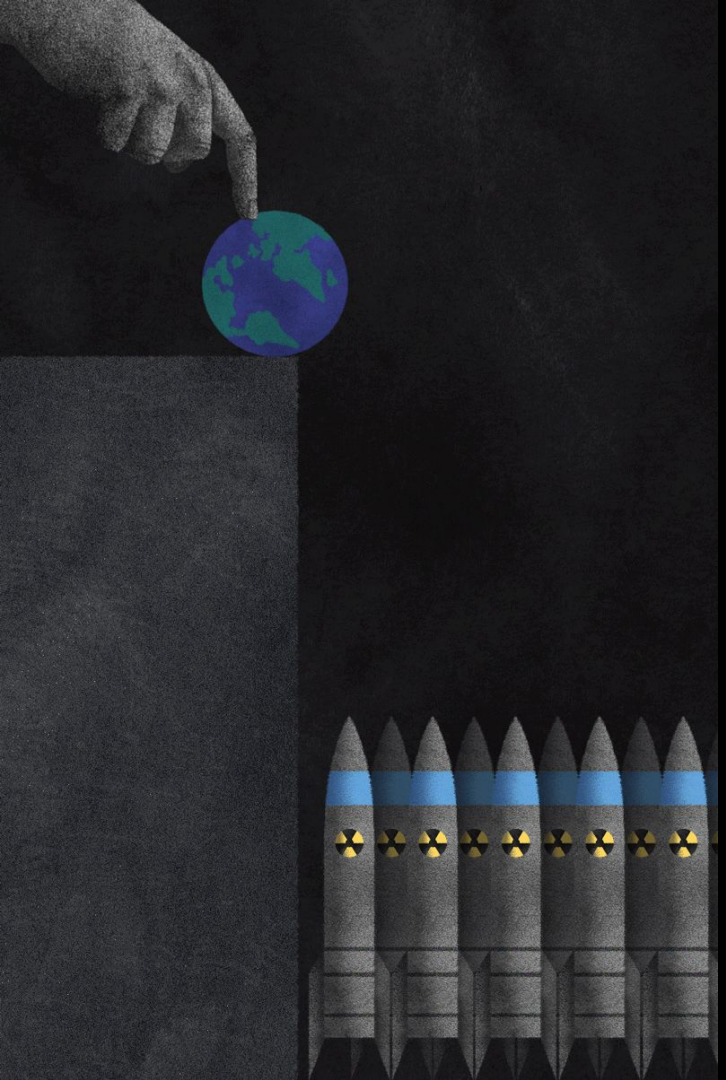

Goldbach's Conjecture: Every even integer greater than 2 can be expressed as the sum of two primes.

Is it true?
Is it false?
Is it undecidable?

Hofstadter explains how all formal systems in which elementary arithmetic can be performed are vulnerable to Gödel's incompleteness theorem.

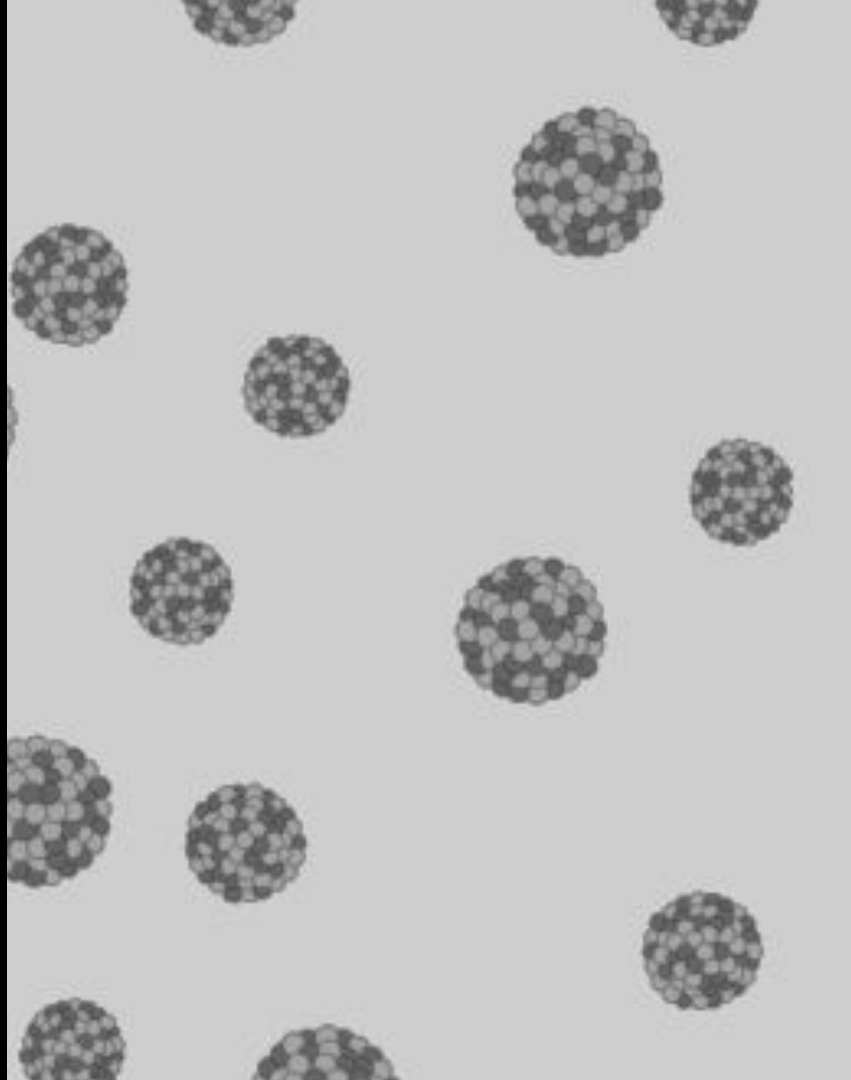


“The fascinating thing is that any such system digs its own hole; the system’s own richness brings about its own downfall. The downfall occurs essentially because the system is powerful enough to have self-referential sentences...”

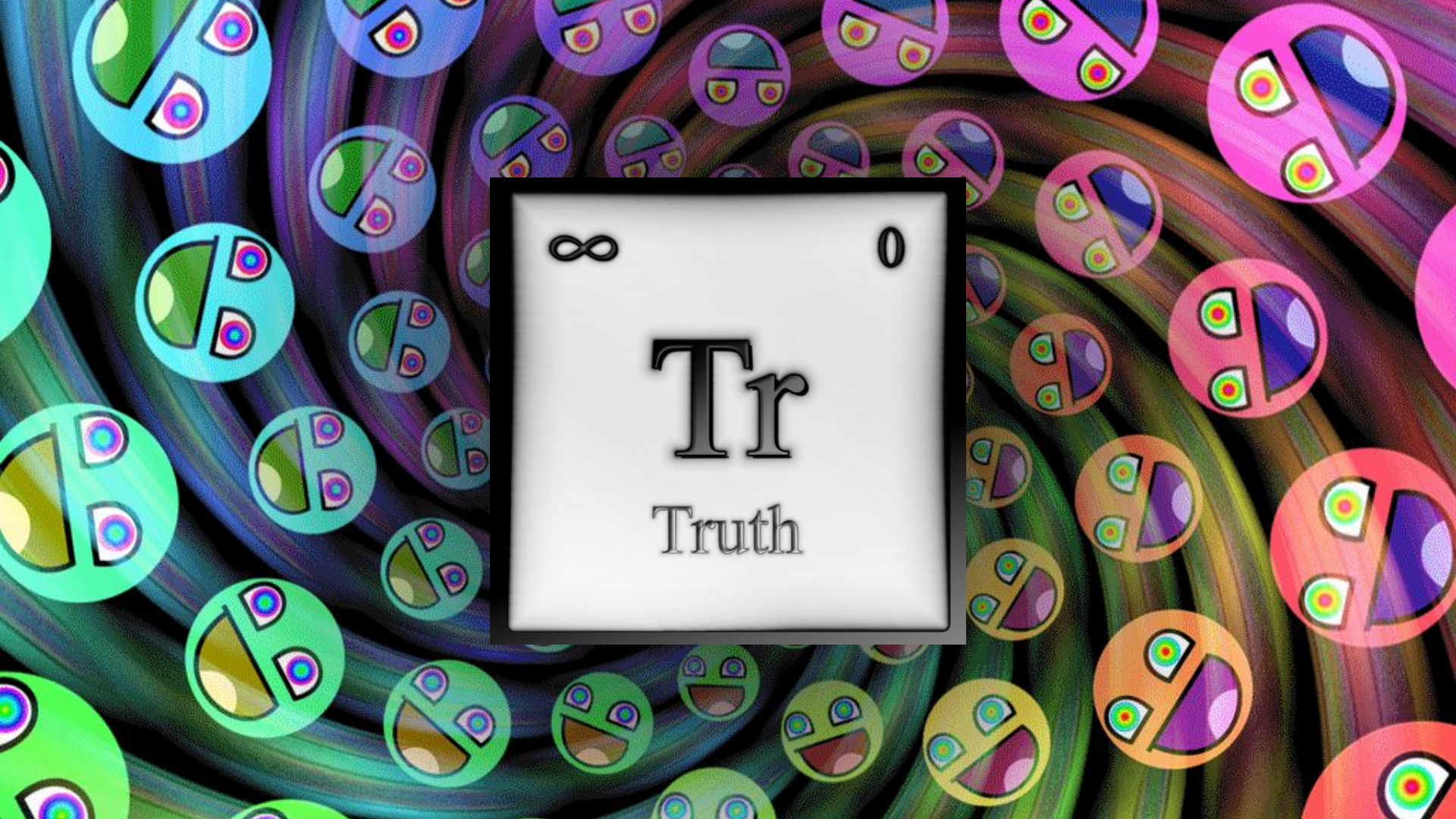


“In physics, the notion exists of a ‘critical mass’ of a fissionable substance, such as uranium. A solid lump of the substance will just sit there, if its mass is less than critical...”

“But beyond the critical mass, such a lump will undergo a chain reaction, and blow up...”



“It seems that with formal systems there is an analogous critical point. Below that point, a system is ‘harmless’... but beyond the critical point, the system suddenly attains the capacity for self-reference, and thereby dooms itself to incompleteness” (Hofstadter 1999: 470).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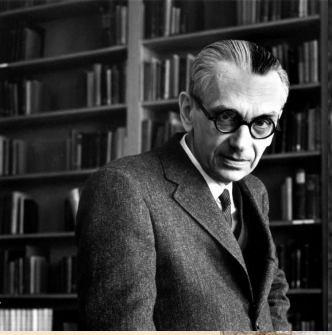
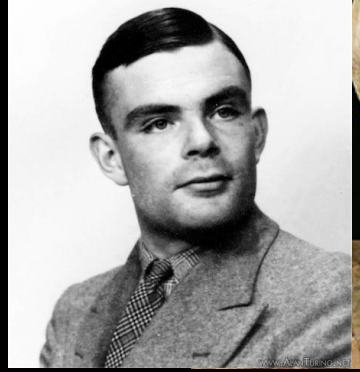
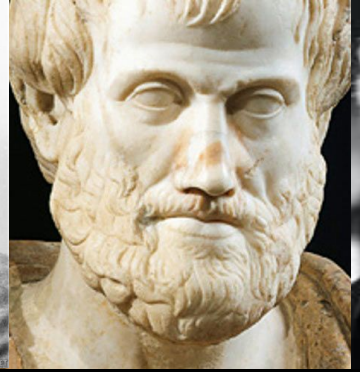
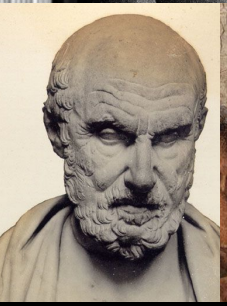
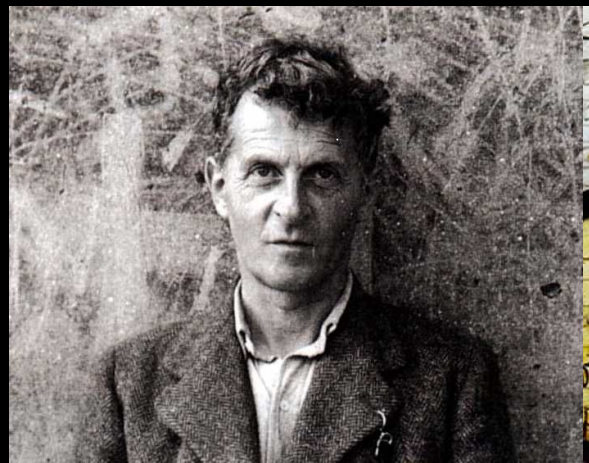


# The Problem of Platonism & Anti-Platonism



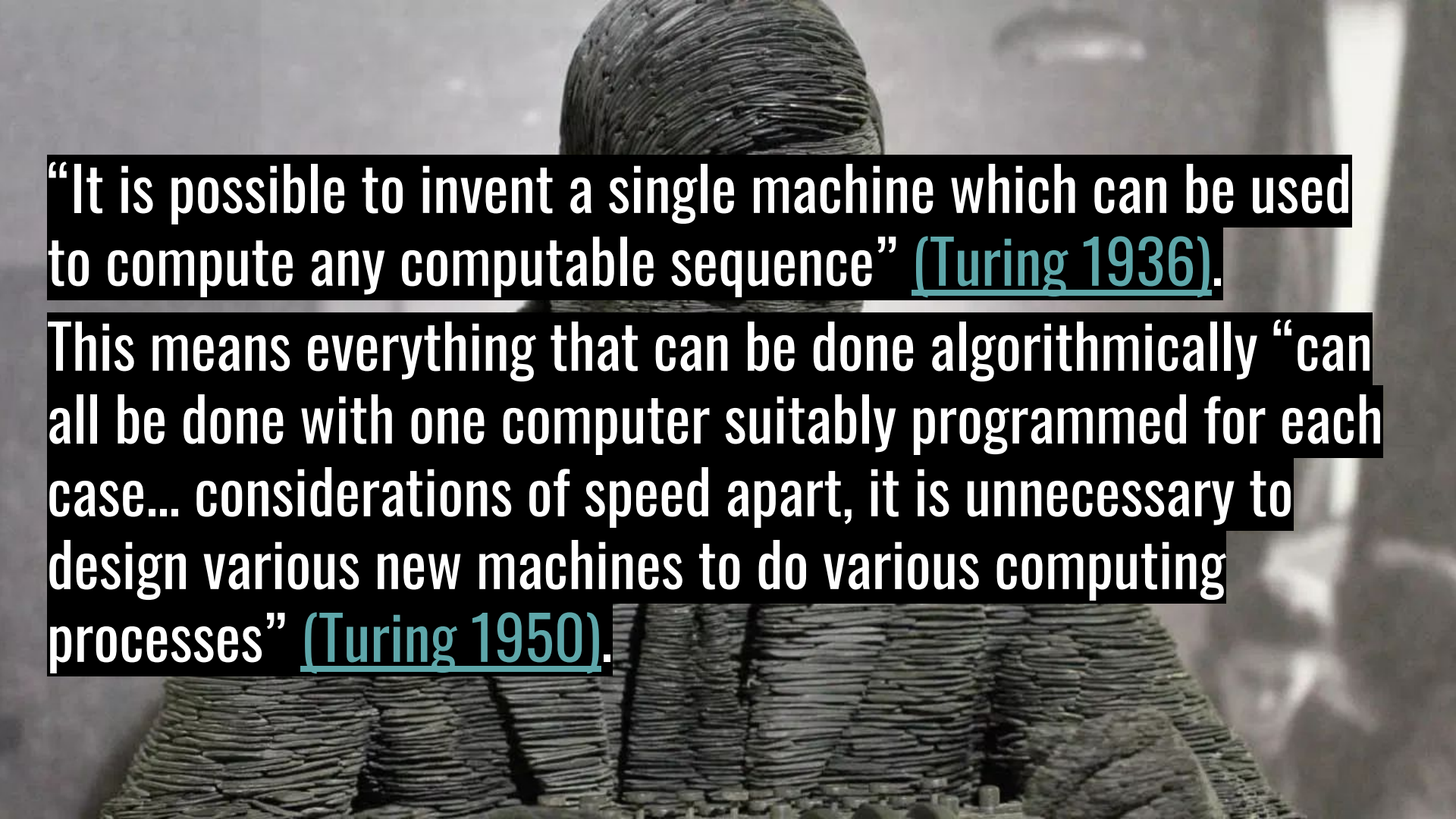






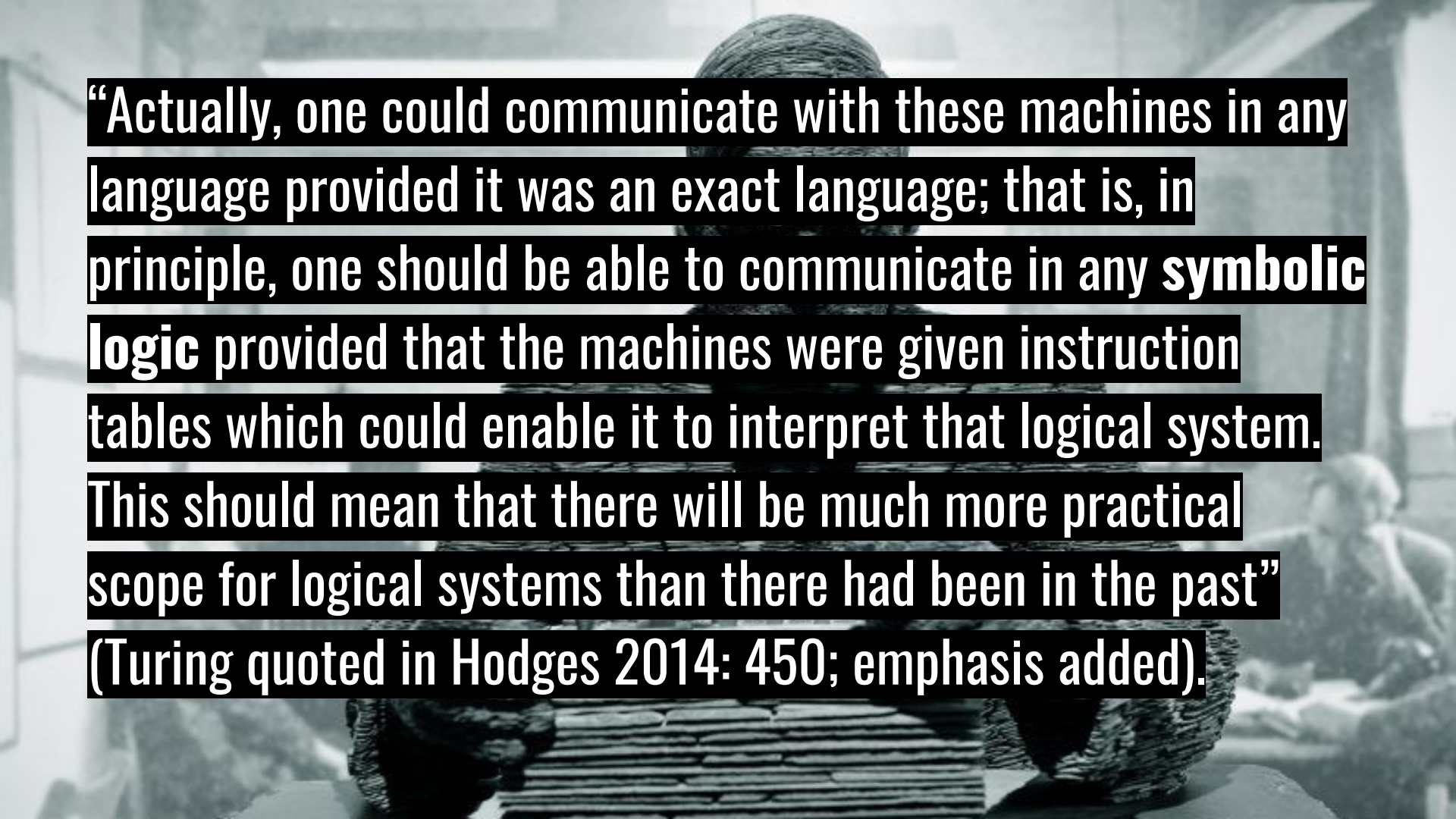
Logicism is false.

# Logic and its connection to computer science...



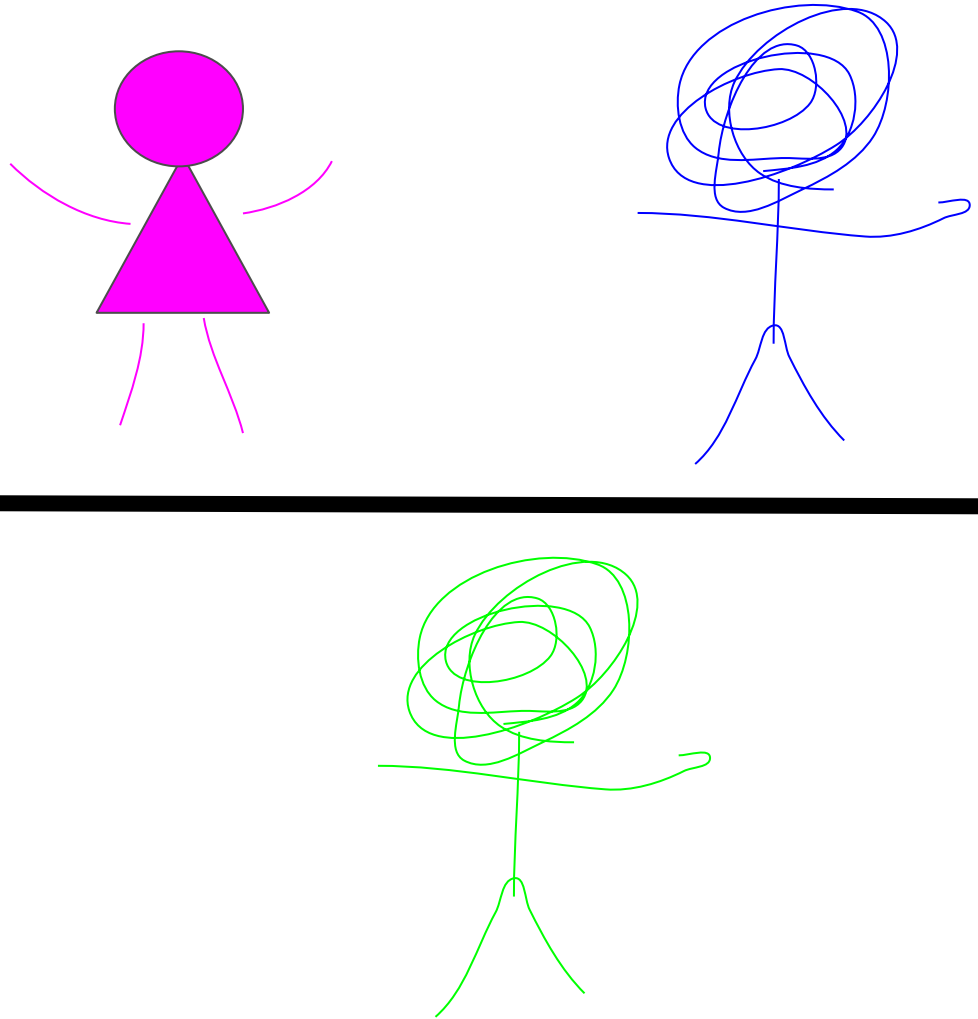
“It is possible to invent a single machine which can be used to compute any computable sequence” (Turing 1936).

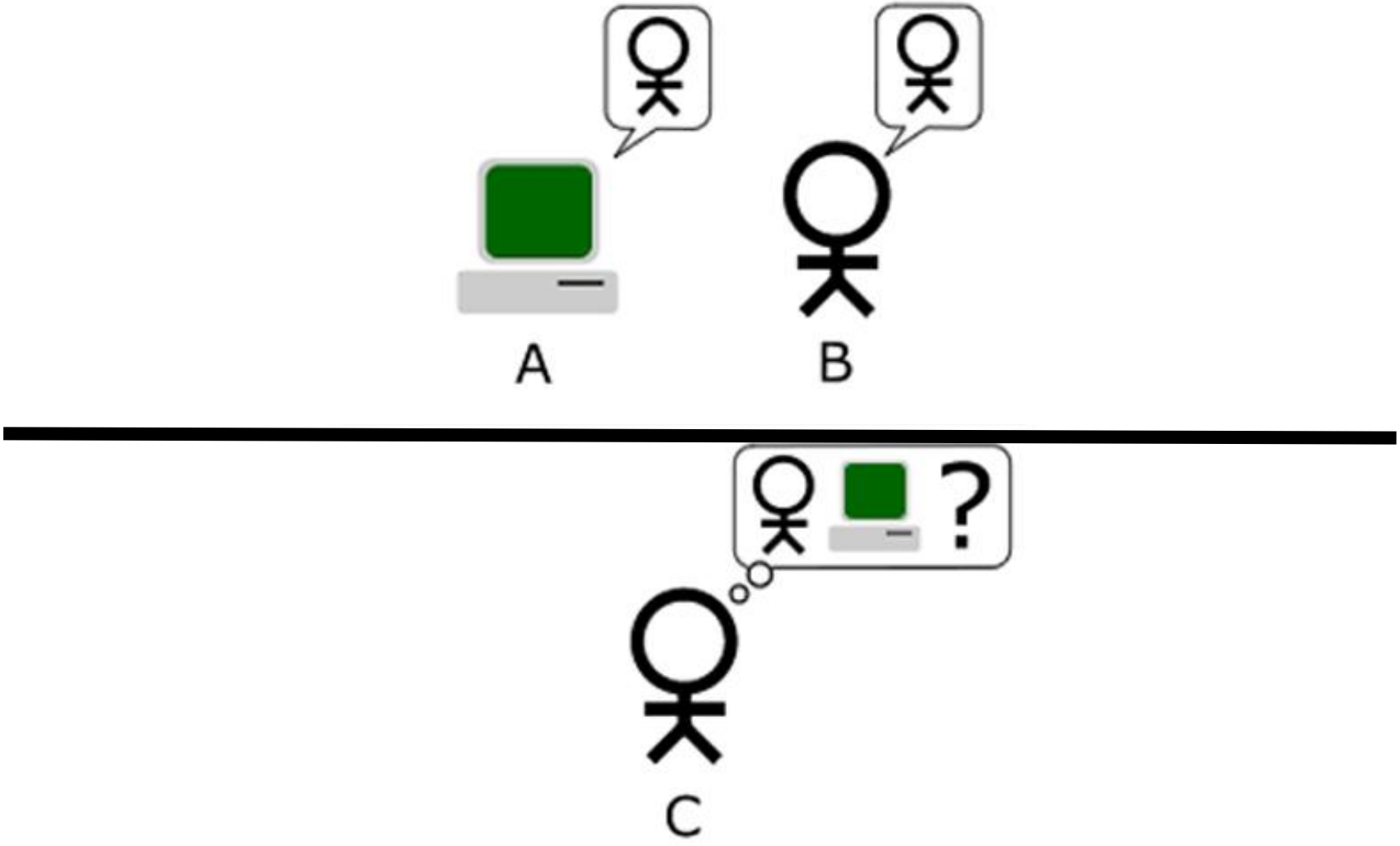
This means everything that can be done algorithmically “can all be done with one computer suitably programmed for each case... considerations of speed apart, it is unnecessary to design various new machines to do various computing processes” (Turing 1950).

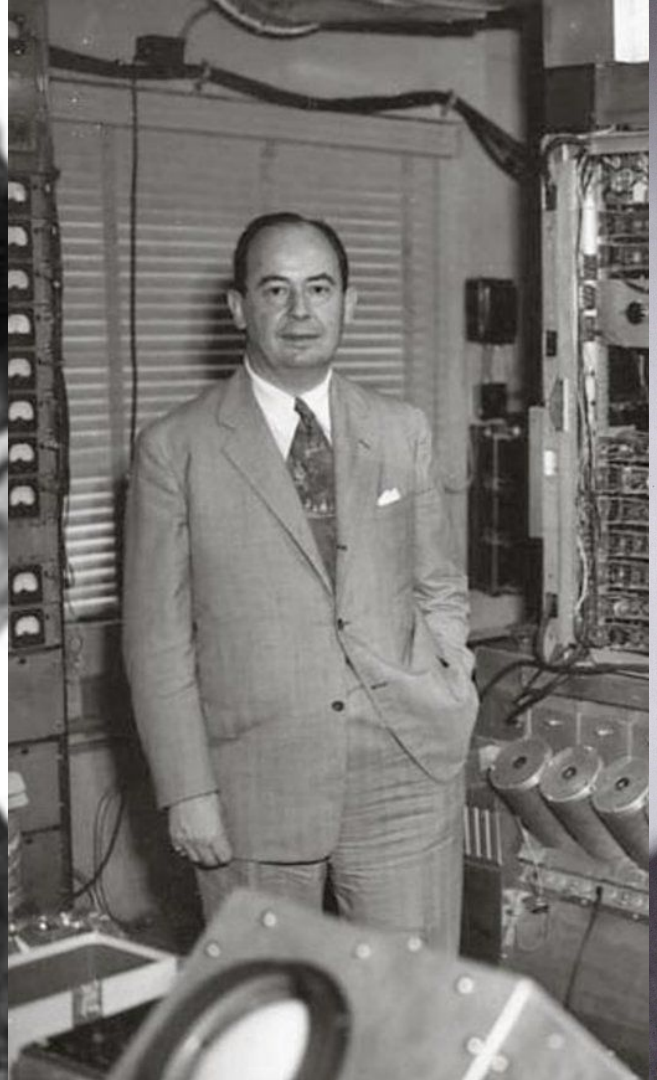


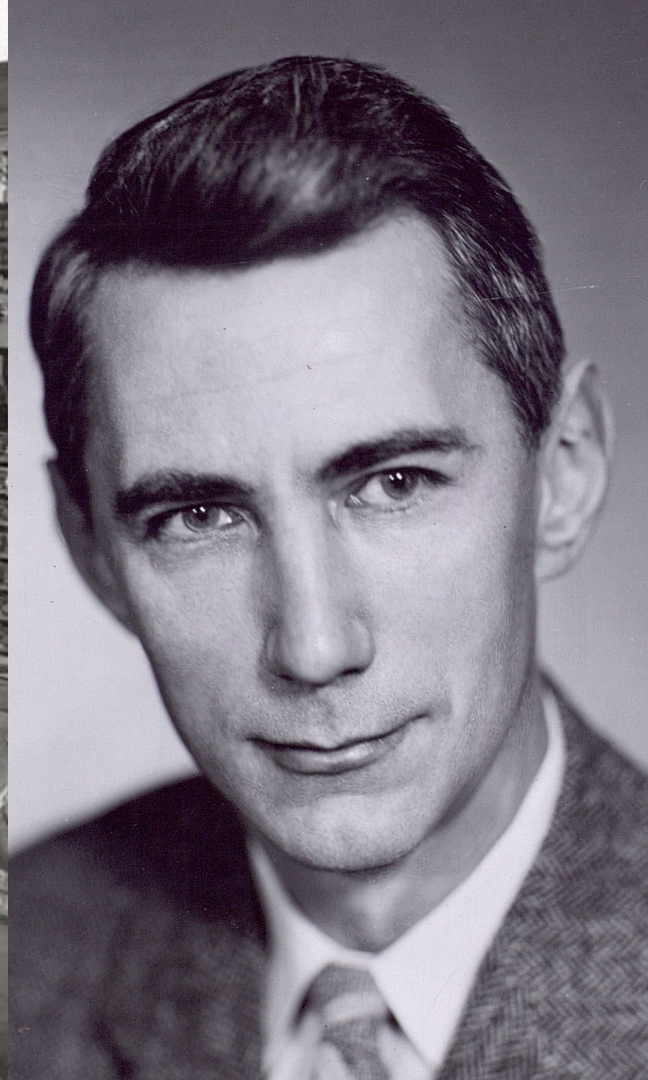
“Actually, one could communicate with these machines in any language provided it was an exact language; that is, in principle, one should be able to communicate in any **symbolic logic** provided that the machines were given instruction tables which could enable it to interpret that logical system. This should mean that there will be much more practical scope for logical systems than there had been in the past” (Turing quoted in Hodges 2014: 450; emphasis added).

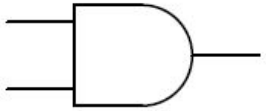
# Logic and its connection to cognitive science...




A
B
?
C

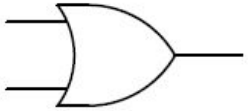






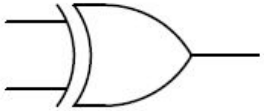
**AND**

| A | B | Output |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |



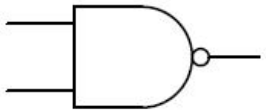
**OR**

| A | B | Output |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |



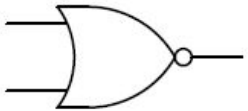
**XOR**

| A | B | Output |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |



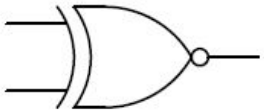
**NAND**

| A | B | Output |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |



**NOR**

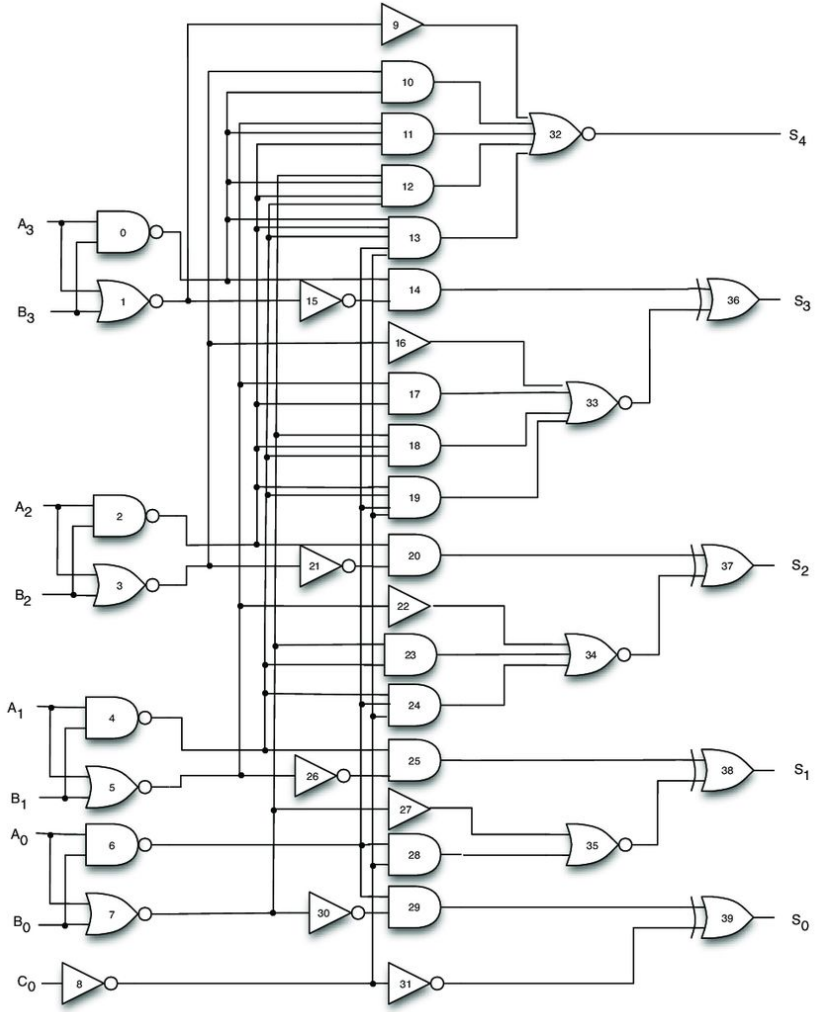
| A | B | Output |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |



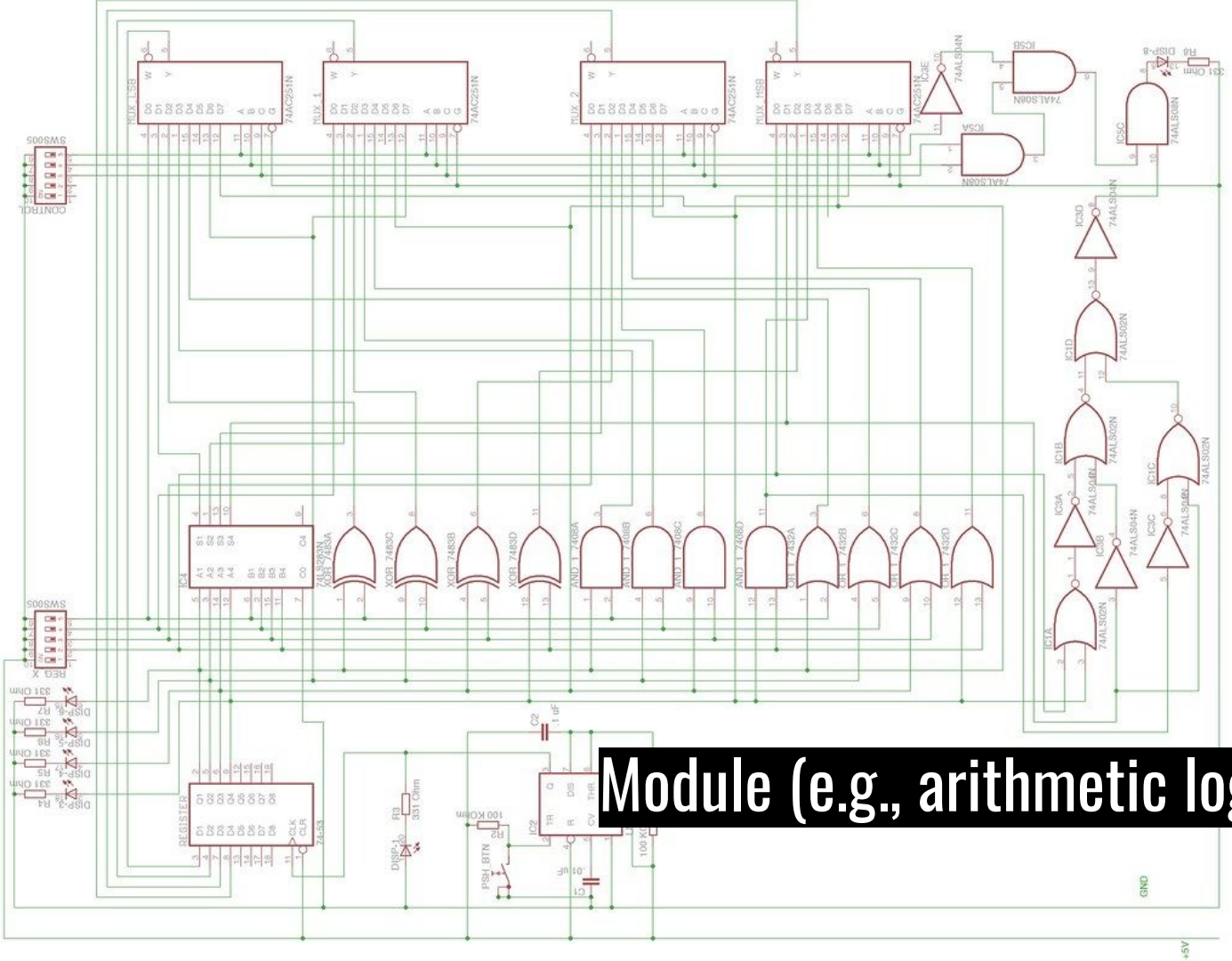
**XNOR**

| A | B | Output |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

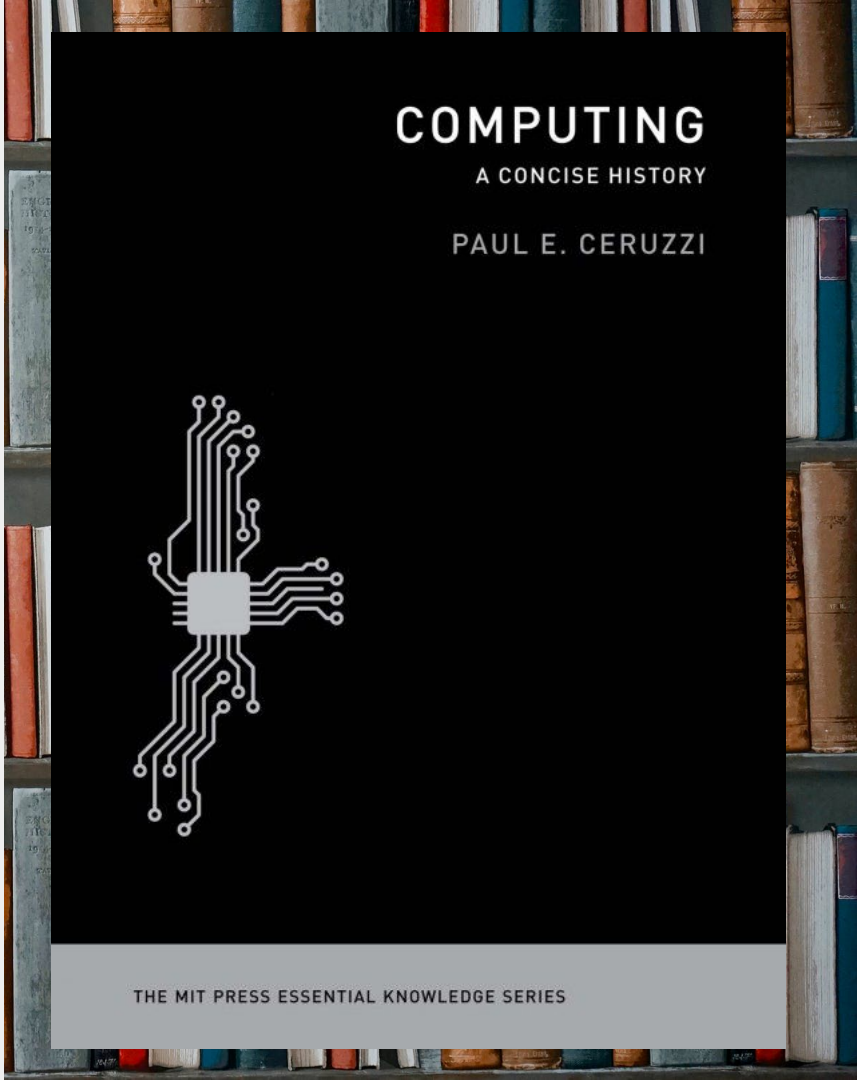
# Logic Gates



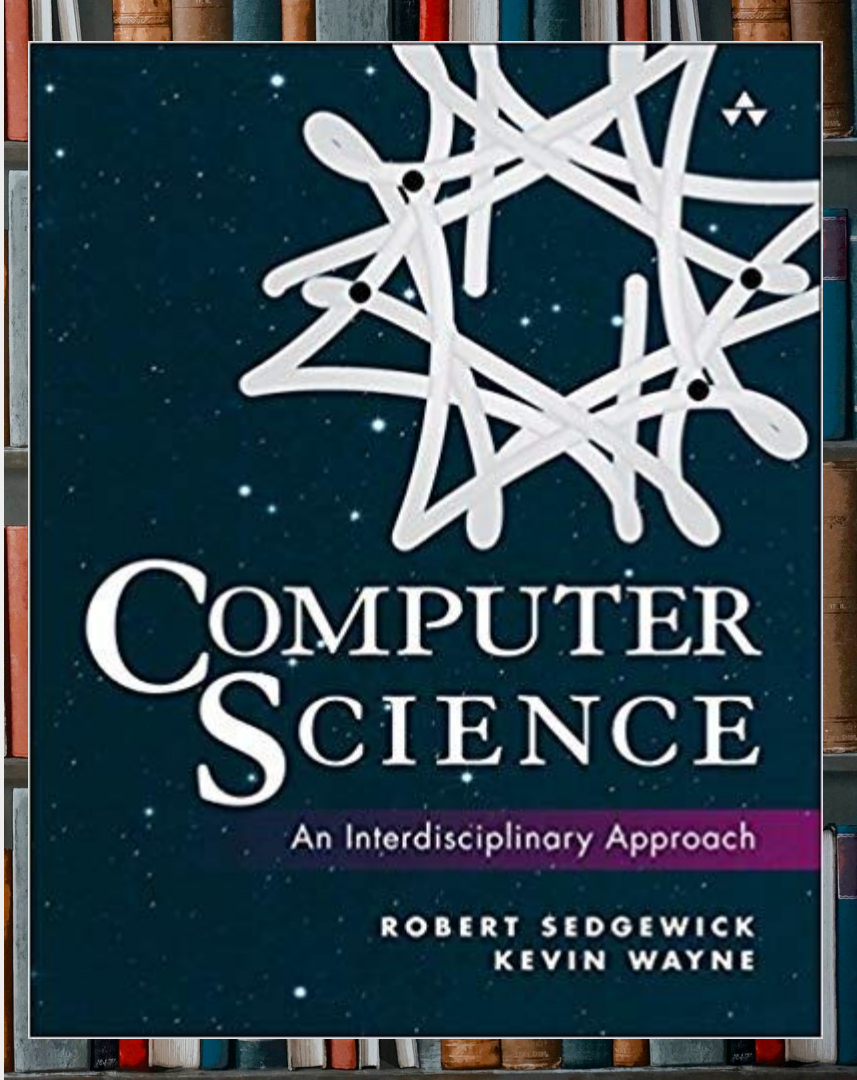

# Gate-level Circuit


Module (e.g., arithmetic logic unit)

Eventually, all the elements of a module were forced onto a single chip, called an **integrated circuit**, or an IC (see Ceruzzi 2012: 86-90).

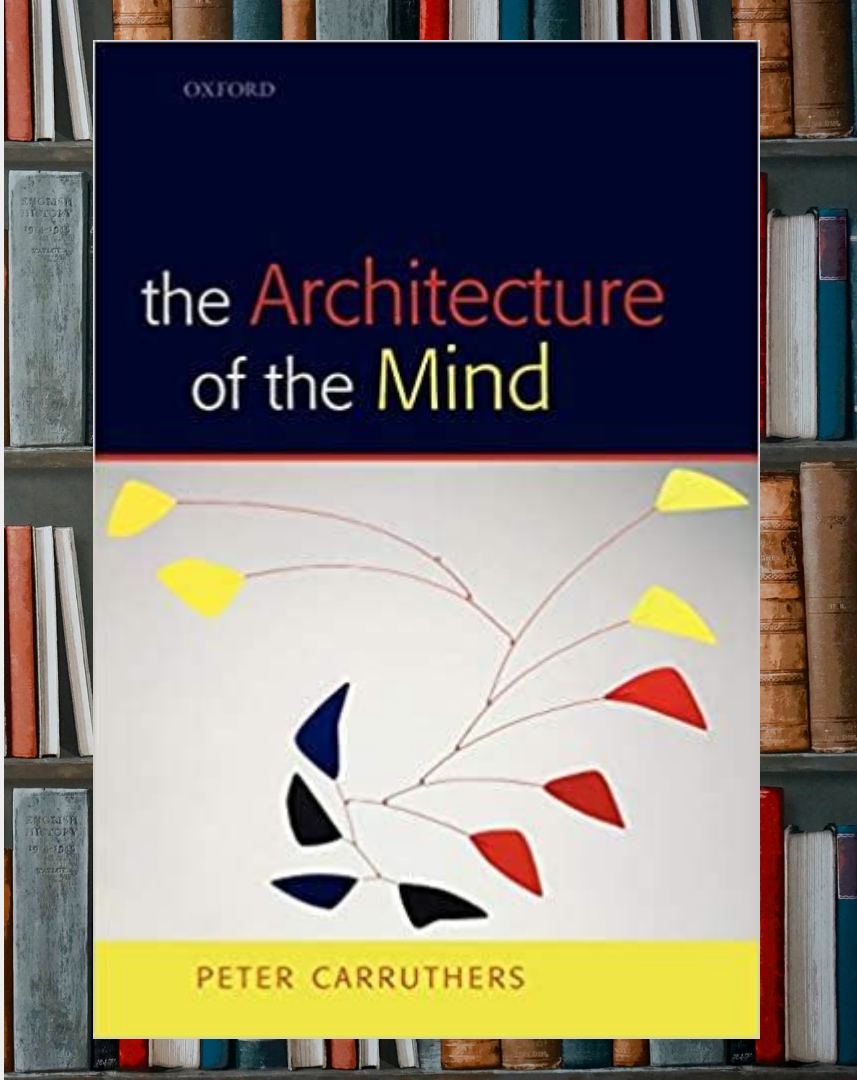


See Sedgewick and Wayne (2016, Chapter 7) for more info.



These same notions are now applied to mental capacities.

This hypothesis is referred to as the **massive modularity of mind** (see Carruthers 2006).

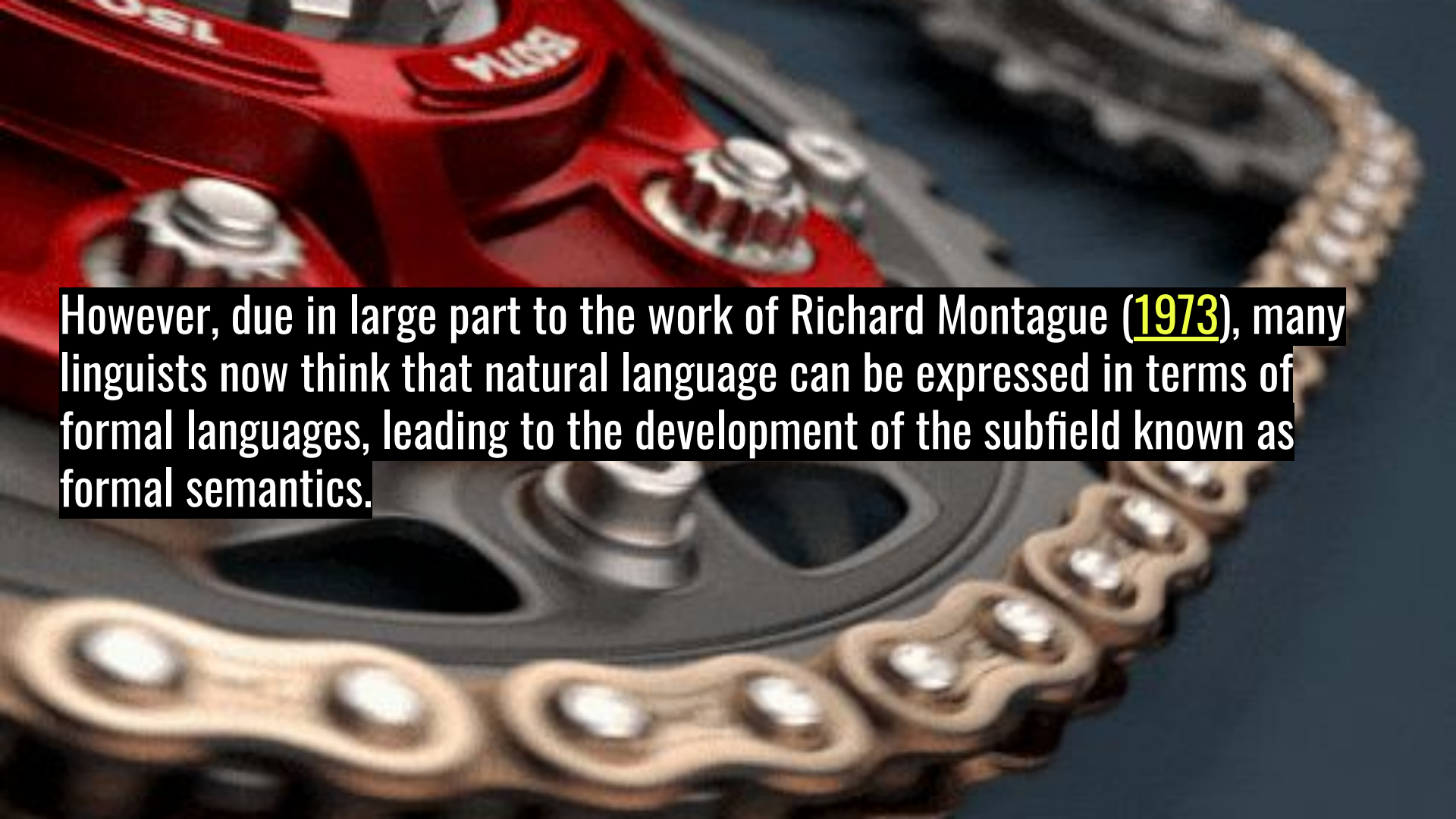


# Logic and its connection to linguistics...

“The development of the predicate calculus and its theory of quantification was one of the triumphs of the modern logical tradition in Europe...

Many of the philosophers and logicians who took part in this development, as I mentioned before, took the view that natural languages were far too obscure, ambiguous, and ill-structured to be amenable to the kind of treatment that they were proposing in their treatment of logic.

And some took the view that they were providing something like an instrument for exhibiting what the true logical form for language *should* be, if it were only well behaved. Thus, in a way, their attitude toward language was revisionist” (Bach 1989: 40).



However, due in large part to the work of Richard Montague (1973), many linguists now think that natural language can be expressed in terms of formal languages, leading to the development of the subfield known as formal semantics.

FIN