CHAPTER THREE

Section 3.1E

1.a. $2^1 = 2$
c. $2^2 = 4$

2.a.

E	↓ ∼	∼	(E	&	∼	E)
T	**F**	**T**	**T**	**F**	**F**	**T**
F	**F**	**T**	**F**	**F**	**T**	**F**

c.

A	J	A	↓ ≡	[J	≡	(A	≡	J)]
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**F**
F	**T**	**F**	**T**	**T**	**F**	**F**	**F**	**T**
F	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**F**

e.

A	H	J	[∼	A	∨	(H	⊃	J)]	↓ ⊃	(A	∨	J)
T	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**T**	**F**	**F**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**F**
T	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**
T	**F**	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**F**
F	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**T**
F	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**F**	**F**	**F**	**F**	**F**
F	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**T**
F	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**F**	**F**	**F**	**F**

g.

A	B	∼	(A	∨	B)	↓ ⊃	(∼	A	∨	∼	B)
T	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**F**	**T**
T	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**F**
F	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**T**
F	**F**	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**T**	**T**	**F**

i.

B	E	H	↓ ∼	(E	&	[H	⊃	(B	&	E)])
T	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**T**	**T**	**T**
T	**F**	**T**	**T**	**F**	**F**	**T**	**F**	**T**	**F**	**F**
T	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**F**
F	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**F**	**F**	**T**
F	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**T**
F	**F**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**F**	**F**
F	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**F**	**F**

k.

D	E	F	~	[D	&	(E	∨	F)]	↓ ≡	[~	D	&	(E	&	F)]
T	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**T**
T	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**F**
T	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**F**	**F**	**T**
T	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**F**
F	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**T**
F	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**T**	**F**	**F**
F	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**F**	**F**	**T**
F	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**F**

m.

A	H	J	(A	∨	(~	A	&	(H	⊃	J)))	↓ ⊃	(J	⊃	H)
T	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**F**	**F**	**T**	**F**	**T**	**T**
T	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**F**
T	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**F**
F	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
F	**T**	**F**	**F**	**F**	**T**	**F**	**F**	**T**	**F**	**F**	**T**	**F**	**T**	**T**
F	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**F**
F	**F**	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**T**	**F**

3. a.

A	B	C	↓ ~	[~	A	∨	(~	C	∨	~	B)]
F	**T**	**T**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**F**	**T**

c.

A	B	C	(A	⊃	B)	↓ ∨	(B	⊃	C)
F	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**

e.

A	B	C	(A	≡	B)	↓ ∨	(B	≡	C)
F	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**T**

g.

A	B	C	~	[B	⊃	(A	∨	C)]	↓ &	~	~	B
F	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**T**

i.

A	B	C	~	[~	(A	≡	~	B)	≡	~	A]	↓ ≡	(B	∨	C)
F	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**T**

Section 3.2E

1.a. Truth-functionally indeterminate

A	~	A	↓ ⊃	A
T	F	T	T	T
F	T	F	F	F

c. Truth-functionally true

A	(A	≡	~	A)	↓ ⊃	~	(A	≡	~	A)
T	T	F	F	T	T	T	T	F	F	T
F	F	F	T	F	T	T	F	F	T	F

e. Truth-functionally indeterminate

B	D	(~	B	&	~	D)	↓ ∨	~	(B	∨	D)
T	T	F	T	F	F	T	F	F	T	T	T
T	F	F	T	F	T	F	F	F	T	T	F
F	T	T	F	F	F	T	F	F	F	T	T
F	F	T	F	T	T	F	T	T	F	F	F

g. Truth-functionally indeterminate

A	B	C	[(A	∨	B)	&	(A	∨	C)]	↓ ⊃	~	(B	&	C)
T	T	T	T	T	T	T	T	T	T	F	F	T	T	T
T	T	F	T	T	T	T	T	T	F	T	T	T	F	F
T	F	T	T	T	F	T	T	T	T	T	T	F	F	T
T	F	F	T	T	F	T	T	T	F	T	T	F	F	F
F	T	T	F	T	T	T	F	T	T	F	F	T	T	T
F	T	F	F	T	T	F	F	F	F	T	T	T	F	F
F	F	T	F	F	F	F	F	T	T	T	T	F	F	T
F	F	F	F	F	F	F	F	F	F	T	T	F	F	F

i. Truth-functionally true

J	K	(J	∨	~	K)	↓ ≡	~	~	(K	⊃	J)
T	T	T	T	F	T	T	T	F	T	T	T
T	F	T	T	T	F	T	T	F	F	T	T
F	T	F	F	F	T	T	F	T	T	F	F
F	F	F	T	T	F	T	T	F	F	T	F

k. Truth-functionally true

A	D	[(A	∨	~	D)	&	~	(A	&	D)]	↓ ⊃	~	D
T	**T**	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**F**	**T**
T	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**F**
F	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**T**
F	**F**	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**F**

2. a. Not truth-functionally true

F	H	(F	∨	H)	↓ ∨	(~	F	≡	H)
F	**F**	**F**	**F**	**F**	**F**	**T**	**F**	**F**	**F**

c. Truth-functionally true

A	B	C	~	A	↓ ⊃	[(B	&	A)	⊃	C]
T	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**F**	**F**
T	**F**	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**T**
T	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**F**
F	**T**	**T**	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**T**
F	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**F**
F	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**T**
F	**F**	**F**	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**F**

e. Truth-functionally true

C	[(C	∨	~	C)	⊃	C]	↓ ⊃	C
T	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**
F	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**F**

3. a.

A	B	A	↓ ⊃	(A	∨	B)
T	**T**	**T**	**T**	**T**	**T**	**T**
T	**F**	**T**	**T**	**T**	**T**	**F**
F	**T**	**F**	**T**	**F**	**T**	**T**
F	**F**	**F**	**T**	**F**	**F**	**F**

c.

A	B	A	↓ ⊃	[B	⊃	(A	&	B)]
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**F**
F	**T**	**F**	**T**	**T**	**F**	**F**	**F**	**T**
F	**F**	**F**	**T**	**F**	**T**	**F**	**F**	**F**

e.

					↓			
A	B	(A	≡	B)	⊃	(A	⊃	B)
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**F**
F	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**T**
F	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**F**

g.

						↓							
A	B	C	(A	⊃	B)	⊃	[(C	⊃	A)	⊃	(C	⊃	B)]
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**T**
T	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**F**
T	**F**	**F**	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**F**
F	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**T**
F	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**F**	**T**	**T**
F	**F**	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**F**	**F**
F	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**T**	**F**

i.

								↓		
A	B	[(A	⊃	B)	&	~	B]	⊃	~	A
T	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**F**	**T**
T	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**T**
F	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**F**
F	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**F**

k.

			↓					
A	B	A	⊃	[B	⊃	(A	⊃	B)]
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**F**
F	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**T**
F	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**F**

m.

						↓							
A	B	D	(A	⊃	B)	⊃	[~	B	⊃	~	(A	&	D)]
T	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**T**
T	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**F**
T	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**T**
T	**F**	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**F**
F	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**T**
F	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**F**
F	**F**	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**F**	**T**
F	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**F**	**F**

o.

A	B	~	(A	≡	B)	↓ ≡	(~	A	≡	B)
T	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**T**
T	**F**	**T**	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**F**
F	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**T**
F	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**F**	**F**

4.a. Truth-functionally false

B	D	(B	≡	D)	↓ &	(B	≡	~	D)
T	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**F**	**T**
T	**F**	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**F**
F	**T**	**F**	**F**	**T**	**F**	**F**	**T**	**F**	**T**
F	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**F**

c. Not truth-functionally false

A	B	A	↓ ≡	(B	≡	A)
T	**T**	**T**	**T**	**T**	**T**	**T**

e. Not truth-functionally false

C	D	[(C	∨	D)	≡	C]	↓ ⊃	~	C
F	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**F**

5.a. False. For example, while ‘(A ⊃ A)’ is truth-functionally true, ‘(A ⊃ A) & A’ is not.

c. True. There cannot be any truth-value assignment on which the antecedent is true and the consequent false because there is no truth-value assignment on which the consequent is false.

e. False. For example, although ‘(A & ~ A)’ is truth-functionally false, ‘C ∨ (A & ~ A)’ is not.

g. True. Since a sentence ~ **P** is false on a truth-value assignment if and only if **P** is true on the truth-value assignment, **P** is truth-functionally true if and only if ~ **P** is truth-functionally false.

i. False. For example, ‘(A ∨ ~ A)’ is truth-functionally true, but ‘(A ∨ ~ A) ⊃ B’ is truth-functionally indeterminate.

6.a. Yes On every truth-value assignment, **P** is true and **Q** is false. Hence **P** ≡ **Q** is false on every truth-value assignment. Therefore **P** ≡ **Q** is truth-functionally false.

c. No. Both ‘A’ and ‘~ A’ are truth-functionally indeterminate, but ‘A ∨ ~ A’ is truth-functionally true.

Section 3.3E

1.a. Not truth-functionally equivalent

A	B	↓ ~	(A	&	B)	↓ ~	(A	∨	B)
T	T	F	T	T	T	F	T	T	T
T	F	T	T	F	F	F	T	T	F
F	T	T	F	F	T	F	F	T	T
F	F	T	F	F	F	T	F	F	F

c. Truth-functionally equivalent

H	K	K	↓ ≡	H	~	K	↓ ≡	~	H
T	T	T	T	T	F	T	T	F	T
T	F	F	F	T	T	F	F	F	T
F	T	T	F	F	F	T	F	T	F
F	F	F	T	F	T	F	T	T	F

e. Truth-functionally equivalent

F	G	(G	⊃	F)	↓ ⊃	(F	⊃	G)	(G	≡	F)	↓ ∨	(~	F	∨	G)
T	T	T	T	T	T	T	T	T	T	T	T	T	F	T	T	T
T	F	F	T	T	F	T	F	F	F	F	T	F	F	T	F	F
F	T	T	F	F	T	F	T	T	T	F	F	T	T	F	T	T
F	F	F	T	F	T	F	T	F	F	T	F	T	T	F	T	F

g. Not truth-functionally equivalent

H	J	K	~	(H	&	J)	↓ ≡	(J	≡	~	K)	(H	&	J)	↓ ⊃	~	K
T	T	T	F	T	T	T	T	T	F	F	T	T	T	T	F	F	T
T	T	F	F	T	T	T	F	T	T	T	F	T	T	T	T	T	F
T	F	T	T	T	F	F	T	F	T	F	T	T	F	F	T	F	T
T	F	F	T	T	F	F	F	F	F	T	F	T	F	F	T	T	F
F	T	T	T	F	F	T	F	T	F	F	T	F	F	T	T	F	T
F	T	F	T	F	F	T	T	T	T	T	F	F	F	T	T	T	F
F	F	T	T	F	F	F	T	F	T	F	T	F	F	F	T	F	T
F	F	F	T	F	F	F	F	F	F	T	F	F	F	F	T	T	F

i. Not truth-functionally equivalent

A	C	D	[A	∨	~	(D	&	C)]	↓ ⊃	~	D	[D	∨	~	(A	&	C)]	↓ ⊃	~	A
T	T	T	T	T	F	T	T	T	F	F	T	T	T	F	T	T	T	F	F	T
T	T	F	T	T	T	F	F	T	T	T	F	F	F	F	T	T	T	T	F	T
T	F	T	T	T	T	T	F	F	F	F	T	T	T	T	T	F	F	F	F	T
T	F	F	T	T	T	F	F	F	T	T	F	F	T	T	T	F	F	F	F	T
F	T	T	F	F	F	T	T	T	T	F	T	T	T	T	F	F	T	T	T	F
F	T	F	F	T	T	F	F	T	T	T	F	F	T	T	F	F	T	T	T	F
F	F	T	F	T	T	T	F	F	F	F	T	T	T	T	F	F	F	T	T	F
F	F	F	F	T	T	F	F	F	T	T	F	F	T	T	F	F	F	T	T	F

k. Not truth-functionally equivalent

F	G	H	F	↓ ∨	~	(G	∨	~	H)	(H	≡	~	F)	↓ ∨	G
T	T	T	T	T	F	T	T	F	T	T	F	F	T	T	T
T	T	F	T	T	F	T	T	T	F	F	T	F	T	T	T
T	F	T	T	T	T	F	F	F	T	T	F	F	T	F	F
T	F	F	T	T	F	F	T	T	F	F	T	F	T	T	F
F	T	T	F	F	F	T	T	F	T	T	T	T	F	T	T
F	T	F	F	F	F	T	T	T	F	F	F	T	F	T	T
F	F	T	F	T	T	F	F	F	T	T	T	T	F	T	F
F	F	F	F	F	F	F	T	T	F	F	F	T	F	F	F

2.a. Truth-functionally equivalent

G	H	G	↓ ∨	H	~	G	↓ ⊃	H
T	T	T	T	T	F	T	T	T
T	F	T	T	F	F	T	T	F
F	T	F	T	T	T	F	T	T
F	F	F	F	F	T	F	F	F

c. Truth-functionally equivalent

A	D	(D	≡	A)	↓ &	D	D	↓ &	A
T	T	T	T	T	T	T	T	T	T
T	F	F	F	T	F	F	F	F	T
F	T	T	F	F	F	T	T	F	F
F	F	F	T	F	F	F	F	F	F

e. Not truth-functionally equivalent

A	A	↓ ≡	(~	A	≡	A)	↓ ~	(A	⊃	~	A)
T	T	F	F	T	F	T	T	T	F	F	T

3.a. Not truth-functionally equivalent

C: The sky clouds over.
N: The night will be clear.
M: The moon will shine brightly.

C	M	N	C	↓ ∨	(N	&	M)	M	↓ ≡	(N	&	~	C)
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**F**	**T**
T	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**F**	**T**
T	**F**	**T**	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**T**
T	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**T**
F	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**
F	**T**	**F**	**F**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**F**
F	**F**	**T**	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**T**	**T**	**F**
F	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**T**	**F**	**F**	**T**	**F**

c. Truth-functionally equivalent

D: The *Daily Herald* reports on our antics.
A: Our antics are effective.

A	D	D	↓ ⊃	A	~	A	↓ ⊃	~	D
T	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**F**	**T**
T	**F**	**F**	**T**	**T**	**F**	**T**	**T**	**T**	**F**
F	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**F**	**T**
F	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**T**	**F**

e. Not truth-functionally equivalent

M: Mary met Tom.
L: Mary liked Tom.
G: Mary asked George to the movies.

G	L	M	(M	&	L)	↓ ⊃	~	G	(M	&	~	L)	↓ ⊃	G
T	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**T**
T	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**T**	**T**
T	**F**	**T**	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**T**
T	**F**	**F**	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**F**	**T**	**F**	**T**	**T**
F	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**F**
F	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**F**	**F**	**T**	**T**	**F**
F	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**F**
F	**F**	**F**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**F**

4.a. Yes. **P** and **Q** have the same truth-value on every truth-value assignment. On every truth-value assignment on which they are both true, ~ **P** and

~ **Q** are both false, and on every truth-value assignment on which they are both false, ~ **P** and ~ **Q** are both true. It follows that ~ **P** and ~ **Q** are truth-functionally equivalent.

c. If **P** and **Q** are truth-functionally equivalent then they have the same truth-value on every truth-value assignment. On those assignments on which they are both true, the second disjunct of ~ **P** ∨ **Q** is true and so is the disjunction. On those assignments on which they are both false, the first disjunct of ~ **P** ∨ **Q** is true and so is the disjunction. So ~ **P** ∨ **Q** is true on every truth-value assignment.

5. No. Although the truth-tables for 'A ∨ B' and 'B ∨ C' have identical columns of truth-values, this does not show that the sentences are truth-functionally equivalent. To establish truth-functional equivalence we need to construct a single truth-table for both of the sentences:

				↓			↓	
A	B	C	A	∨	B	B	∨	C
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**F**
T	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**T**
T	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**F**
F	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**
F	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**F**
F	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**T**
F	**F**	**T**	**F**	**F**	**F**	**F**	**F**	**F**

As rows 4 and 7 show, 'A ∨ B' and 'B ∨ C' are not truth-functionally equivalent. Constructing separate truth-tables for these sentences rather than one table for both sentences does not show that these sentences have the same truth-values on *all* combinations of truth-values that their *combined* atomic components can have, and hence does not establish truth-functional equivalence.

6.a.

			↓					↓			
P	Q	P	≡	Q	(P	⊃	Q)	&	(Q	⊃	P)
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**F**	**T**	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**T**
F	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**F**
F	**F**	**F**	**T**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**F**

c.

				↓							↓			
P	Q	R	P	&	(Q	∨	R)	(P	&	Q)	∨	(P	&	R)
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**F**	**F**
T	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**T**
T	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**F**
F	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**F**	**T**
F	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**F**
F	**F**	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**F**	**F**	**F**	**T**
F	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**F**

e.

P	Q	↓ ~	(P	≡	Q)	(~	P	↓ ≡	Q)
T	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**T**
T	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**F**
F	**T**	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**T**
F	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**F**	**F**

Section 3.4E

1.a. Truth-functionally consistent

A	B	C	A	↓ ⊃	B	B	↓ ⊃	C	A	↓ ⊃	C
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**F**
T	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**T**	**T**
T	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**F**
F	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**T**
F	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**F**
F	**F**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**T**
F	**F**	**F**	**F**	**T**	**F**	**F**	**T**	**F**	**F**	**T**	**F**

c. Truth-functionally inconsistent

H	J	L	↓ ~	[J	∨	(H	⊃	L)]	L	↓ ≡	(~	J	∨	~	H)	H	↓ ≡	(J	∨	L)
T	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**T**
T	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**F**
T	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**T**
T	**F**	**F**	**T**	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**F**	**F**	**F**
F	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**T**
F	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**F**
F	**F**	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**F**	**F**	**F**	**F**	**T**	**T**
F	**F**	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**F**

e. Truth-functionally inconsistent

H	J	(J	⊃	J)	↓ ⊃	H	↓ ~	J	↓ ~	H
T	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**T**
T	**F**	**F**	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**T**
F	**T**	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**F**
F	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**F**

g. Truth-functionally consistent

			↓	↓	↓
A	B	C	A	B	C
T	**T**	**T**	**T**	**T**	**T**
T	**T**	**F**	**T**	**T**	**F**
T	**F**	**T**	**T**	**F**	**T**
T	**F**	**F**	**T**	**F**	**F**
F	**T**	**T**	**F**	**T**	**T**
F	**T**	**F**	**F**	**T**	**F**
F	**F**	**T**	**F**	**F**	**T**
F	**F**	**F**	**F**	**F**	**F**

i. Truth-functionally consistent

						↓				↓	↓
A	B	C	(A	&	B)	∨	(C	⊃	B)	~ A	~ B
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F T**	**F T**
T	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**F T**	**F T**
T	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**F**	**F**	**F T**	**T F**
T	**F**	**F**	**T**	**F**	**F**	**T**	**F**	**T**	**F**	**F T**	**T F**
F	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**T F**	**F T**
F	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**T**	**T**	**T F**	**F T**
F	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**F**	**F**	**T F**	**T F**
F	**F**	**F**	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**T F**	**T F**

2.a. Truth-functionally consistent

				↓					↓	
B	D	E	B	⊃	(D	⊃	E)	~ D	&	B
T	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**T F**	**T**	**T**

c. Truth-functionally consistent

				↓					↓	
F	J	K	F	⊃	(J	∨	K)	F	≡	~ J
T	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T F**

e. Truth-functionally consistent

					↓				↓
A	B	(A	⊃	B)	≡	(~ B	∨	B)	A
T	**T**	**T**	**T**	**T**	**T**	**F T**	**T**	**T**	**T**

3.a. Truth-functionally inconsistent

S: Space is infinitely divisible.
Z: Zeno's paradoxes are compelling.
C: Zeno's paradoxes are convincing.

				↓		↓				↓
C	S	Z	S	⊃	Z	~	(C	∨	Z)	S
T	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**T**
T	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**T**
T	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**T**	**F**
T	**F**	**F**	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**F**
F	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**T**
F	**T**	**F**	**T**	**F**	**F**	**T**	**F**	**F**	**F**	**T**
F	**F**	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**F**
F	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**F**	**F**	**F**

c. Truth-functionally consistent

E: Eugene O'Neill was an alcoholic.
P: Eugene O'Neill's plays show that he was an alcoholic.
I: *The Iceman Cometh* must have been written by a teetotaler.
F: Eugene O'Neill was a fake.

				↓	↓	↓		↓	
E	F	I	P	E	P	I	E	∨	F
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**

e. Truth-functionally consistent

R: The Red Sox will win next Sunday.
J: Joan bet $5.00 against the Red Sox.
E: Joan will buy Ed a hamburger.

				↓						& ↓		
E	J	R	R	⊃	(J	⊃	E)	~	R	&	~	E
F	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**F**

4.a. First assume that {**P**} is truth-functionally inconsistent. Then, since **P** is the only member of {**P**}, there is no truth-value assignment on which **P** is true; so **P** is false on every truth-value assignment. But then ~ **P** is true on every truth-value assignment, and so ~ **P** is truth-functionally true.

Now assume that ~ **P** is truth-functionally true. Then ~ **P** is true on every truth-value assignment, and so **P** is false on every truth-value assignment. But then there is no truth-value assignment on which **P**, the only member of {**P**}, is true, and so the set is truth-functionally inconsistent.

c. No. For example, 'A' and '~ A' are both truth-functionally indeterminate, but {A, ~ A} is truth-functionally inconsistent.

Section 3.5E

1.a. Truth-functionally valid

				↓					↓						
A	H	J	A	⊃	(H	&	J)	J	≡	H	~	J	~	A
T	T	T	T	T	T	T	T	T	T	T	F	T	F	T
T	T	F	T	F	T	F	F	F	F	T	T	F	F	T
T	F	T	T	F	F	F	T	T	F	F	F	T	F	T
T	F	F	T	F	F	F	F	F	T	F	T	F	F	T
F	T	T	F	T	T	T	T	T	T	T	F	T	T	F
F	T	F	F	T	T	F	F	F	F	T	T	F	T	F
F	F	T	F	T	F	F	T	T	F	F	F	T	T	F
F	F	F	F	T	F	F	F	F	T	F	T	F	T	F

c. Truth-functionally valid

							↓										↓				↓		
A	D	G	(D	≡	~	G)	&	G	(G	∨	[(A	⊃	D)	&	A])	⊃	~	D	G	⊃	~	D
T	T	T	T	F	F	T	F	T	T	T	T	T	T	T	T	F	F	T	T	F	F	T
T	T	F	T	T	T	F	F	F	F	T	T	T	T	T	T	F	F	T	F	T	F	T
T	F	T	F	T	F	T	T	T	T	T	T	F	F	F	T	T	T	F	T	T	T	F
T	F	F	F	F	T	F	F	F	F	F	T	F	F	F	T	T	T	F	F	T	T	F
F	T	T	T	F	F	T	F	T	T	T	F	T	T	F	F	F	F	T	T	F	F	T
F	T	F	T	T	T	F	F	F	F	F	F	T	T	F	F	T	F	T	F	T	F	T
F	F	T	F	T	F	T	T	T	T	T	F	T	F	F	F	T	T	F	T	T	T	F
F	F	F	F	F	T	F	F	F	F	F	F	T	F	F	F	T	T	F	F	T	T	F

e. Truth-functionally valid

						↓				↓		↓	
C	D	E	(C	⊃	D)	⊃	(D	⊃	E)	D	C	⊃	E
T	T	T	T	T	T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T	F	F	T	T	F	F
T	F	T	T	F	F	T	F	T	T	F	T	T	T
T	F	F	T	F	F	T	F	T	F	F	T	F	F
F	T	T	F	T	T	T	T	T	T	T	F	T	T
F	T	F	F	T	T	F	T	F	F	T	F	T	F
F	F	T	F	T	F	T	F	T	T	F	F	T	T
F	F	F	F	T	F	T	F	T	F	F	F	T	F

g. Truth-functionally valid

					↓										↓				
G	H	(G	≡	H)	∨	(~	G	≡	H)	(~	G	≡	~	H)	∨	~	(G	≡	H)
T	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**T**
T	**F**	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**F**	**F**
F	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**T**
F	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**F**

i. Truth-functionally invalid

					↓						↓				↓	
F	G	~	~	F	⊃	~	~	G	~	G	⊃	~	F	G	⊃	F
T	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**T**	**T**
T	**F**	**T**	**F**	**T**	**F**	**F**	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**T**
F	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**F**
F	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**F**

2. a. Truth-functionally valid

					↓						↓					↓	
J	M	(J	∨	M)	⊃	~	(J	&	M)	M	≡	(M	⊃	J)	M	⊃	J
T	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**F**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**F**	**F**	**F**	**T**	**T**	**F**	**T**	**T**
F	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**F**	**F**
F	**F**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**F**	**F**	**F**	**T**	**F**	**F**	**T**	**F**

c. Truth-functionally valid

			↓						↓			↓		
A	B	A	⊃	~	A	(B	⊃	A)	⊃	B	A	≡	~	B
T	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**T**
T	**F**	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**F**
F	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**T**
F	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**F**	**T**	**F**

e. Truth-functionally invalid

				↓										↓				↓	
A	B	C	A	&	~	[(B	&	C)	≡	(C	⊃	A)]	B	⊃	~	B	~	C	⊃	C
T	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**F**	**F**	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**F**

3.a. Truth-functionally valid

					↓			
B	C	(B	&	C)	⊃	(B	∨	C)
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**F**
F	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**T**
F	**F**	**F**	**F**	**F**	**T**	**F**	**F**	**F**

c. Truth-functionally invalid

														↓					
J	T	([(J	⊃	T)	⊃	J]	&	[(T	⊃	J)	⊃	T])	⊃	(~	J	∨	~	T)
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**T**

e. Truth-functionally invalid

										↓	
B	C	D	[(B	&	C)	&	(B	∨	D)]	⊃	D
T	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**F**

4.a. Truth-functionally invalid

S: ‘Stern’ means the same as ‘star’.
N: ‘Nacht’ means the same as ‘day’.

N ⊃ S
~ N
———
~ S

			↓		↓		↓	
N	S	N	⊃	S	~	N	~	S
F	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**T**

c. Truth-functionally invalid

S: Sophie is in her right mind.
J: Jason is in his right mind.
T: Sophie believes in trolls.
R: Jason believes in trolls

(S ⊃ ~ T) & S
(J ⊃ ~ R) & ~ J
———
~ T & R

								↓						↓					↓	
S	J	T	R	(S	⊃	~	T)	&	S	(J	⊃	~	R)	&	~	J	~	T	&	R
T	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**F**

e. Truth-functionally valid

D: Computers can have desires.
E: Computers can have emotions.
T: Computers can think.

D	E	T	T ≡ E (↓ ≡)			E ⊃ D (↓ ⊃)			D ⊃ ~ T (↓ ⊃)			~ T (↓ ~)
			T	≡	E	E	⊃	D	D	⊃	~ T	~ T
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**F T**	**F T**
T	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T F**	**T F**
T	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**F T**	**F T**
T	**F**	**F**	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**T F**	**T F**
F	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**F T**	**F T**
F	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T F**	**T F**
F	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**F**	**F**	**T**	**F T**	**F T**
F	**F**	**F**	**F**	**T**	**F**	**F**	**T**	**F**	**F**	**T**	**T F**	**T F**

5. a. If {**P**} ⊨ **Q**, then there is no truth-value assignment on which **P** is true and **Q** is false. If {**Q**} ⊨ **P**, then there is no truth-value assignment on which **Q** is true and **P** is false. So there is no truth-value assignment on which **P** and **Q** have different truth-values, and therefore they are truth-functionally equivalent. Conversely, assume that **P** and **Q** are truth-functionally equivalent. Then **Q** is true on every truth-value assignment on which **P** is true, so {**P**} ⊨ **Q**; and **P** is true on every truth-value assignment on which **Q** is true, so {**Q**} ⊨? **P**.

c. Assume that {**P**} ⊨ **Q** and {**Q**} ⊨ **R**. Then, by the first entailment, if **P** is true on a truth-value assignment, **Q** is also true on that assignment. By the second entailment, **R** must be true on that truth-value assignment as well. Therefore {**P**} ⊨ **R**.

Section 3.6E

1.a. If {~ **P**} is truth-functionally inconsistent, then there is no truth-value assignment on which ~ **P** is true (since ~ **P** is the only member of its unit set). But then ~ **P** is false on every truth-value assignment, so **P** is true on every truth-value assignment and is truth-functionally true.

c. If Γ ∪ {~ **P**} is truth-functionally inconsistent, then there is no truth-value assignment on which every member of Γ ∪ {~ **P**} is true. But ~ **P** is true on a truth-value assignment if and only if **P** is false on that assignment. Hence there is no truth-value assignment on which every member of Γ is true and **P** is false. Hence Γ ⊨ **P**.

2.a. **P** is truth-functionally true if and only if the set {~ **P**} is truth-functionally inconsistent. But {~**P**} is the same set as ∅ ∪ {~ **P**}. So **P** is truth-functionally true if and only if ∅ ∪ {~ **P**} is truth-functionally inconsistent. But we have already seen, by previous results, that ∅ ∪ {~ **P**} is truth-functionally inconsistent if and only if ∅ ⊨ **P**. Hence **P** is truth-functionally true if and only if ∅ ⊨ **P**.

c. Assume that Γ is truth-functionally inconsistent. Then there is no truth-value assignment on which every member of Γ is true. Let **P** be an *arbitrarily* selected sentence of *SL*. Then there is no truth-value assignment on which every member of Γ is true and **P** false since there is no truth-value assignment on which every member of Γ is true. Hence Γ ⊨ **P**.

3.a. Since Γ is a truth-functionally consistent set there is at least one truth-value assignment on which every member of Γ is true. But **P** is also true on such an assignment since a truth-functionally true sentence is true on every truth-value assignment. Hence on at least one truth-value assignment every member of Γ ∪ {**P**} is true; so the set is truth-functionally consistent.

4.a. **P** is either true or false on each truth-value assignment. On any assignment on which **P** is true, **Q** is true (because {**P**} ⊨ **Q**) and so **Q** ∨ **R** is true. On any assignment on which **P** is false, ~ **P** is true, **R** is therefore also true (because {~ **P**} ⊨ **R**), and so **Q** ∨ **R** is true as well. Either way, then, **Q** ∨ **R** is true—so the sentence is truth-functionally true.

c. Assume that every member of Γ ∪ Γ′ is true on some truth-value assignment. Then every member of Γ is true, and so **P** is true (because Γ ⊨ **P**). Every member of Γ′ is also true, and so **Q** is true (because Γ′ ⊨ **Q**). Therefore **P** & **Q** is true. So Γ ∪ Γ′ ⊨ **P** & **Q**.