Chapter 2

SYNTAX AND SYMBOLIZATION

Section 2.1 of this chapter presents the formal language *SL* ('*SL*' is short for 'Sentential Logic'). Section 2.2 introduces the symbolization process, that is, how English sentences are symbolized in *SL*. Section 2.3 is devoted to developing proficiency in the symbolization process. Section 2.4 explores some of the complexities involved in symbolizing English sentences in *SL*.

2.1 THE SYNTAX OF *SL*

The **syntax** of a language specifies the basic expressions of a language and the rules that determine which combinations of those expressions count as sentences of the language. The syntax of a language does not specify how the sentences of the language are to be interpreted; that is a matter for **semantics**, which we will address in Chapter 3. The syntax of English, and every other natural language, is enormously complex. Fortunately, the syntax of *SL* is simple, straightforward, and easily learned.

But before we lay out the syntax of *SL* we need to introduce some terminology.

METALANGUAGE/OBJECT LANGUAGE

Throughout the rest of this text we will be using English to talk about two formal languages, first *SL* and later *PL*. When we use a language to talk about a language, we are using that language as a **metalanguage,** and the language that we are talking about is the **object language.** So when we are talking about *SL* and *PL* they are the object languages.[1]

USE AND MENTION

We regularly use language to talk about or of a host of different things,

> . . . of shoes—and ships—and sealing-wax—Of cabbages—and kings—

as Lewis Carroll wrote. But we also have occasion to use language to talk about language. In this text we will frequently talk of expressions, sentences, and arguments of *SL* (and later of *PL*), as well as words, sentences, and arguments of English. When we talk about these or other linguistic constructions, large or small, we are **mentioning** rather than **using** those constructions and it is important that we have ways of indicating that we are doing so. Failure to indicate that we are mentioning rather than using a piece of language can lead to confusion. Consider the sentence:

> Minnesota derives from a Native American word.

We can probably all figure out what a person who asserts this sentence means, namely, that the name of the state located between the Dakotas and Wisconsin derives from a Native American word. But what the sentence literally says is that Minnesota, the state, a political entity, derives from a Native American word, and this is clearly false.

In this text, we use two conventions to indicate that we are mentioning or talking about language. The first is to place the linguistic expression we are mentioning within single quotation marks. So we can make the intended claim about the origin of the name of Minnesota by saying that 'Minnesota' derives from a Native American word. The second convention we will use is that of displaying the language we wish to talk about or mention on an indented line or lines. Thus we can truly say that the following sentence is about the origin of the name of Minnesota:

> 'Minnesota' derives from a Native American word.

We have just said something about a sentence, and we indicated that we were doing so by displaying that sentence on a line by itself. Within the displayed sentence, we used the convention of placing an expression that we are talking about within single quotes. We have used both of these conventions earlier in this text, and we will use them throughout the rest of this text.

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[1]In a German class the instructor uses English to talk about German, and in this instance English is the metalanguage and German is the object language. And when a grammar instructor uses English to talk about the rules of English grammar English is both the metalanguage and the object language.

METAVARIABLES

Most of us are familiar with mathematicians' use of the letters 'x' and 'y' to make arithmetic claims such as

> For any positive integers x and y, if x is even and y is odd then x + y is odd.

Of course 'x' and 'y' are not integers. They are letters of the English alphabet. But they are used by mathematicians to make general claims about, in this case, integers. The letters 'x' and 'y' when so used are said to be **variables** and to **range over** or **take as values** the positive integers, that is, the numbers 1, 2, 3, 4 . . .

Analogously to the way mathematicians use 'x' and 'y' as variables ranging over numbers we will use the boldface capital letters '**P**', '**Q**', '**R**', and '**S**', with or without subscripts, as in

$\mathbf{P}$ $\mathbf{P}_1$ $\mathbf{Q}_3$

as **metavariables** ranging over expressions of the object languages *SL* and *PL*. These variables are termed 'metavariables' because they are parts of the metalanguage we are using, English, not parts of the object languages *SL* and *PL*. We will similarly use the boldfaced lowercase letters '**p**', '**q**', '**r**', and '**s**', with or without following primes, as metavariables ranging over expressions of English.

We can now lay out the syntax of *SL*. We begin by specifying the expressions or **vocabulary** of *SL*. These are

Sentence Letters: the capital Roman letters 'A' through 'Z', with or without positive integer subscripts[2]:

A, B, C, . . . , A_1, B_1, C_1, . . . , A_2, B_2, C_2, . . .

Sentential Connectives:

~	(called the 'tilde')
&	(called the 'ampersand')
$\vee$	(called the 'wedge')
$\supset$	(called the 'horseshoe')
$\equiv$	(called the 'triple bar')

[2]The inclusion of capital Roman letters with positive integer subscripts among the sentence letters of *SL* means that there are infinitely many sentence letters of *SL*. This is appropriate as there are infinitely many claims that can be made about the universe and its contents and we never know how many of these claims someone may want to symbolize by using sentence letters of *SL*.

The connective tilde is a **unary connective**; the remaining connectives are **binary connectives.** Binary connectives connect, as the name suggests, two sentences to form a new sentence. The unary connective tilde attaches to a single sentence to form a new sentence.[3]

Punctuation marks: '(' and ')'

Recursive Definition of 'Sentence of *SL*'

We define '**sentence of *SL***' as follows:

1. Every sentence letter of *SL* is a sentence of *SL*.
2. If **P** is a sentence of *SL*, then ~ **P** is a sentence of *SL*.
3. If **P** and **Q** are sentences of *SL*, then (**P** & **Q**) is a sentence of *SL*.
4. If **P** and **Q** are sentences of *SL*, then (**P** ∨ **Q**) is a sentence of *SL*.
5. If **P** and **Q** are sentences of *SL*, then (**P** ⊃ **Q**) is a sentence of *SL*.
6. If **P** and **Q** are sentences of *SL*, then (**P** ≡ **Q**) is a sentence of *SL*.
7. Nothing is a sentence of *SL* unless it can be formed by repeated application of clauses 1–6.[4]

Our specification of the syntax of *SL* is now complete. Our recursive definition of 'sentence of *SL*' provides a complete specification of what expressions counts as a sentence of *SL*.

All of the following expressions are sentences of *SL*, as we shall explain:

(B & D)

((B ≡ D) ∨ ~ C)

~ ~ D

((A & B) & ~ (C ≡ ~ D))

'(B & D)' contains two sentence letters, 'B' and 'D'. By clause 1, they are both sentences of *SL*. Since they are sentences of *SL*, '(B & D)' is also, by clause

[3]Expressions that attach to a single sentence to form a new sentence, as does the tilde, are traditionally, though somewhat misleadingly, termed 'connectives' though they do not connect two sentences.

[4]Readers are unlikely to have difficulty understanding clauses 2–6 of our recursive definition of 'sentence of *SL*. For example, the import of clause 3

If **P** and **Q** are sentences of *SL*, then (**P** & **Q**) is a sentence of *SL*,

is simply that the result of placing '&' between any two sentences of *SL* and enclosing the result in parentheses is a sentence of *SL*. But there is a complexity here that we note for the sake of completeness. We stipulate that in expressions that contain both metavariables and expressions of *SL* the metavariables are being used and the expressions of *SL* are being mentioned. We need this stipulation because in claims such as clauses 2–6 of our recursive definition we are *talking about* what expressions constitute sentences of *SL*, not using sentences of *SL*. We adopt a parallel convention for hybrid expressions in which we use the metalinguistic variables '**p**', '**q**', '**r**', and/or '**s**' to refer to expressions of English.

3, a sentence of *SL*. The second listed sentence contains the sentence letters 'B', 'C', and 'D'. These are all sentences of *SL* by clause 1. Therefore '(B ≡ D)' is a sentence of *SL*, by clause 6, and '~ C' is a sentence of *SL*, by clause 2. Since '(B & D)' and '~ C' are both sentences of *SL*, '((B ≡ D) ∨ ~ C)' is also a sentence of *SL*, by clause 4.

Turning to the third sentence, 'D' is a sentence of *SL,* by clause 1. Therefore, by clause 2, '~ D' is also a sentence of *SL*, and since '~ D' is a sentence of *SL*, so is '~ ~ D', again by clause 2. The fourth listed sentence contains four sentence letters, 'A', 'B', 'C', and 'D' and these are all sentences of *SL* by clause 1. Since 'A' and 'B' are sentences of *SL* by clause 1 so is '(A & B)', by clause 3. And since 'D' is a sentence of *SL*, so is '~ D', by clause 2. Because 'C' and '~ D' are both sentences of *SL*, '(C ≡ ~ D)' is a sentence of *SL*, by clause 6. It follows that '~ (C ≡ ~ D)' is a sentence of *SL* by clause 2. Finally, it follows by clause 3 that '((A & B) & ~ (C ≡ D)' is a sentence of *SL*.

The following expressions are *not* sentences of *SL*:

B & D
∨ A
(BC ⊃ D)
(B ⊂ (C ∨ D))
(**P** ≡ **Q**)

- 'B & D' is not a sentence of *SL* because the only clause of our definition that introduces an ampersand is clause 3 and it requires that when sentences are joined by an ampersand the result be placed within parentheses. 'B & D' contains no parentheses, so it is not a sentence of *SL*. (However, we shall adopt an *informal* convention of allowing the deletion of outer parentheses, so that 'B & D' will count informally as a sentence of *SL*. All parentheses other than outer parentheses are necessary to make it clear what sentences binary connectives are connecting.)
- '∨ A' contains a wedge, and the only clause that introduces a wedge is clause 4, which requires a sentence in front of the wedge as well as a sentence after the wedge. So '∨ A' is not a sentence of *SL*.
- '(BC ⊃ D)' contains a horseshoe, and clause 5 is the only clause that introduces a horseshoe. But for clause 5 to be applicable, 'BC' would have to be a sentence of *SL*. It is not, because there is no clause in our definition that allows two sentences of *SL* to be concatenated without placing a connective between them. So '(BC ⊃ D)' is not a sentence of *SL*.

- '(B ⊂ (C ∨ D))' is not a sentence of *SL* because '⊂' is not a symbol of *SL*.
- '(**P** ≡ **Q**)' is not a sentence because neither '**P**' nor '**Q**' is a sentence of *SL*. The sentence letters of *SL* do not include boldface letters.

We complete this section by laying out terminology associated with the syntax of *SL*. First, our definition of 'sentence of *SL*' is a **recursive definition**. Recursive definitions start by directly identifying some items that the concept being defined applies to. In our case clause 1 of our definition specifies that the concept 'sentence of *SL*' applies to the sentence letters of *SL*. Subsequent clauses specify that if one or more items are such that the concept in question applies to them, then that concept applies to some additional item. In our definition, clauses 2 through 6 do this. These clauses say that if some expression or expressions are sentences of *SL* then so is another expression. Our recursive definition of 'sentence of *SL*' ends with a closure clause, which says that there is nothing else the concept being defined applies to.

Sentences of *SL* are of two sorts: **atomic sentences** and **compound sentences**. The sentence letters of *SL* constitute the atomic sentences of *SL*. (They are called 'atomic sentences' because they are not formed or compounded from other sentences.) All non-atomic sentences are **compound** sentences, so called because they are formed or compounded from other sentences of *SL*. All compound sentences contain at least one sentential connective. There are five types of compound sentences and each type has a **main connective** and an **immediate component** or **components**:

Negations: sentences of the form ~ **P**. The *main connective* of ~ **P** is '~' and **P** is the immediate component.

Conjunctions: sentences of the form (**P** & **Q**). The *main connective* of (**P** & **Q**) is '&' and **P** and **Q** are the immediate components.

Disjunctions: sentences of the form (**P** ∨ **Q**). The *main connective* of (**P** ∨ **Q**) is '∨' and **P** and **Q** are the immediate components.

Material Conditionals: sentences of the form (**P** ⊃ **Q**). The *main connective* of (**P** ⊃ **Q**) is '⊃' and **P** and **Q** are the immediate components.

Material Biconditionals: sentences of the form (**P** ≡ **Q**). The *main connective* of (**P** ≡ **Q**) is '≡' and **P** and **Q** are the immediate components.

The immediate components of a conjunction are the **conjuncts** of that conjunction and the immediate components of a disjunction are the **disjuncts** of that disjunction. The immediate components of a material conditional are the **antecedent**—which precedes the main connective—and the **consequent**—which follows the main connective.

Below we list several examples of each kind of truth-functional compound sentence of *SL*. The arrows point to the main connectives of these sentences.

All of the following are *negations* of *SL*:

↓

$\sim A$

↓

$\sim (B \,\&\, C)$

↓

$\sim (A \vee (D \,\&\, B))$

↓

$\sim \sim (A \equiv D)$

All of the following are *conjunctions* of *SL*:

↓

$(A \,\&\, B)$

↓

$(A \,\&\, (B \vee C))$

↓

$((C \,\&\, \sim D) \,\&\, (\sim D \vee B))$

↓

$(D \,\&\, ((C \equiv A) \vee \sim (A \supset B)))$

All of the following are *disjunctions* of *SL*:

↓

$(D \vee \sim A)$

↓

$(B \vee (A \supset \sim D))$

↓

$((B \equiv \sim D) \vee (A \,\&\, \sim C))$

↓

$(\sim \sim (A \,\&\, \sim D) \vee (B \supset (A \equiv \sim D)))$

All of the following are *material conditionals* of *SL*:

↓

$(A \supset B)$

↓

$((B \,\&\, \sim C) \supset \sim D)$

↓

$(\sim D \supset (B \,\&\, (C \vee \sim A)))$

↓

$((A \equiv \sim B) \supset (B \supset \sim A))$

All of the following are *material biconditionals* of *SL*:

↓
(~ D ≡ ~ A)

↓
(B ≡ (~ A & ~ C))

↓
((~ A & ~ B) ≡ (C ∨ ~ D))

↓
((C ≡ ~ D) ≡ ~ A)

We next introduce the notion of a **component** of a sentence of *SL*.

The *components* of a sentence **P** of *SL* are

- **P** itself,
- The immediate components (if any) of **P**,
- The components of **P**'s immediate components.

What this comes to is that every sentence that can be found within **P**, as well as **P** itself, counts as a component of **P**. This is how it works. Consider the compound sentence

~ (A ⊃ (B & ~ D))

By definition, this sentence is a component of itself. The immediate component of this negation,

(A ⊃ (B & ~ D))

is also by definition a component of the negation. The two immediate components of this sentence:

A

(B & ~ D)

are therefore components of '(A ⊃ (B & ~ D))' and therefore of '~ (A ⊃ (B & ~ D))', and so on. By this definition, the components of '~ (A ⊃ (B & ~ D))' are

~ (A ⊃ (B & ~ D))

(A ⊃ (B & ~ D))

A

(B & ~ D)

B

~ D

D

Finally, we establish two conventions that make it easier to work with sentences of *SL*. First, we informally allow the deletion of **outermost parentheses** as deleting them does not introduce any ambiguity as to what sentences binary connectives are connecting. A sentence of *SL* that begins with a left parenthesis and ends with a right parenthesis has outermost parentheses, and they can, by our convention, be omitted. So we can write 'A ⊃ (B & ~ D)' rather than '(A ⊃ (B & ~ D))'. Note that this convention about outermost parentheses does not apply to negations of compound sentences. (Negations do not have outermost parentheses; they start with a '~', not a left parentheses.) For example, '~ ((A ∨ B) ⊃ ~ (C ≡ D))' cannot be rewritten as '~ (A ∨ B) ⊃ ~ (C ≡ D)'. The former is a negation and the latter is a material conditional. We also allow the use of square brackets ('[' and ']') in place of parentheses to make complicated sentences easier to read. For example, if we write '~ ((A ∨ B) ⊃ ((A ⊃ ~ C) ≡ D))' as '~ [(A ∨ B) ⊃ [(A ⊃ ~ C) ≡ D]]', it becomes easier to discern the structure of the sentence.

2.1**E** EXERCISES

1. Which of the following are sentences of *SL* and which are not? For those that are not, explain why they are not.

a. & H
*b. B & Z
c. ~ O
*d. M ~ N
e. J ⊃ (K ⊃ (A ∨ N))
*f. **P** ∨ **Q**
g. (I ∨ [T & E])
*h. (U & C & ~ L)
i. [(G ∨ E) ⊃ (~ H & (K ⊃ B)]
*j. (F ≡ K) ⊃ [M ∨ K]

2. For each of the following sentences, indicate whether the sentence is a negation, a conjunction, a disjunction, a material conditional, or a material biconditional.

a. A ⊃ B
*b. ~ A ∨ B
c. ~ A ≡ ~ B
*d. ~ ~ (A ⊃ B)
e. ~ A ⊃ (B & ~ D)
*f. (D ≡ ~ A) ≡ B
g. ~ (A ≡ B) & (~ C ⊃ D)
*h. ~ ~ ~ B
i. [A & ~ (B ∨ C)] ⊃ [(A & ~ B) & (A & ~ C)]
*j. (A ⊃ B) & (B ⊃ A)
k. ~ (~ A ⊃ ~ B)
*l. ~ A ⊃ B
m. ~ ~ (A ⊃ B) ∨ (C ⊃ D)
*n. (A ∨ ~ B) ⊃ ~ (C & ~ D)

3. For each of the following sentences, circle the main connective and underline the immediate sentential component(s). Then list all the sentential components, including the atomic components.

a. ~ A & H
*b. ~ (A & H)
c. ~ (S & G) ∨ B
*d. K ⊃ (~ K ⊃ K)
e. (C ≡ K) ⊃ [~ H ⊃ (M & N)]
*f. M ⊃ [~ N ⊃ ((B & C) ≡ ~ [(L ⊃ J) ∨ X])]

4. Which of the following characters can occur immediately to the left of '~' in a sentence of *SL*? When one can so occur, give a sentence of *SL* in which it does; when it cannot so occur, explain why. Which of these characters can occur immediately to the right of 'A' in a sentence of *SL*? When one can so occur, give a sentence of *SL* in which it does; when it cannot so occur, explain why.

a. H
*b. &
c. (
*d.)
e. [
*f. ~

2.2 INTRODUCTION TO SYMBOLIZATION

As we have seen, the sentence letters or atomic sentences of *SL* can be combined using connectives and parentheses to form compound sentences of considerable complexity. But what is the relation between sentences of *SL* and English sentences? The sentence letters of *SL* can be used to symbolize English sentences. In theory, any sentence letter of *SL* can symbolize any English sentence. Recall the simple argument we used as an example in Chapter 1:

Either the maid or the butler killed Watson.
If it was the maid, Watson was poisoned.
Watson wasn't poisoned.
____
The butler killed Watson.

For convenience, we will refer to this English language argument as our "whodunit". We could use 'A' to symbolize the first premise, 'B' to symbolize the second, 'C' to symbolize the third, and 'D' to symbolize the conclusion. Our symbolic argument would then be

A
B
C
—
D

Our whodunit is clearly valid. But the premises of our *symbolic* argument provide no apparent support for the conclusion, and that argument will turn out to be an invalid argument of *SL*. The problem is that this symbolic argument does not reflect the structure of the English language argument. That is, there are important relationships among the premises and conclusion that are not reflected in our symbolization of our whodunit. To capture those relationships, we need to use compound sentences of *SL* to symbolize the premises and conclusion of our whodunit. To this end, we need to know how the sentential connectives of *SL* are to be interpreted. The following list pairs connectives of *SL* with expressions of English to which they roughly correspond.

~	It is not the case that . . .
&	. . . and . . .
∨	. . . or . . .
⊃	if . . . then . . .
≡	. . . if and only if . . .

Given this information about the connectives, a far better symbolization of our whodunit is

M ∨ B
M ⊃ W
~ W
———
B

We can specify the English sentences that we are symbolizing with the sentence letters 'M', 'B', and 'W' as follows:

M: The maid killed Watson.
B: The Butler killed Watson.
W: Watson was poisoned.

We call such specifications '**symbolization keys**', and we will use them throughout this chapter. A symbolization key for a group of atomic sentences of *SL* allows us to construct English readings for sentences of *SL* that contain those sentence letters. For example, an appropriate English reading of '~ (M & B)' given our current symbolization key is

It is not the case that both the maid and the butler killed Watson,

or, more colloquially,

The maid and the butler didn't both kill Watson.

Note that the first premise of our whodunit, 'Either the maid or the butler killed Watson' is not literally a compound sentence, that is, it does not consist of two sentences connected by 'or'. But it is clearly equivalent to such a compound sentence, namely

> The maid killed Watson or the butler killed Watson.

Similarly, 'The butler and the maid both hated Watson' is not a compound sentence consisting of two English sentences connected by 'and'; but it is equivalent to the compound sentence 'The butler hated Watson and the maid hated Watson'. If we expand our current symbolization key to include:

> H: The butler hated Watson
> A: The maid hated Watson

we can symbolize this additional information about the maid and the butler as 'H & A'.

When we symbolize English sentences, we usually choose sentence letters of *SL* that may help us to remember which sentences they are symbolizing. For example, we earlier used 'M' to symbolize the sentence 'The maid killed Watson' in the expectation that using 'M' will help us remember that 'M' is symbolizing a sentence about the maid. In symbolizing our whodunit we selected 'B' and 'W' for analogous reasons. There is no formal requirement that sentence letters be correlated in this manner with the sentences that they symbolize, and when we are using one symbolization key to symbolize a significant number of sentences, it often becomes impossible to use this mnemonic device. It is, however, a requirement that each sentence letter symbolize only one sentence in a given symbolization key. So we cannot expand the previous whodunit symbolization key to include 'M' as a symbolization of 'The maid hated Watson' because in that symbolization key 'M' is already used to symbolize 'The maid killed Watson'. Once we selected 'H' to symbolize 'The butler hated Watson' no obvious mnemonic letter is available to symbolize 'The maid hated Watson'. Hence we arbitrarily chose the letter 'A'.

It is time to make the process of symbolizing English sentences somewhat more systematic. We have already seen that when the sentences to be symbolized are compound sentences whose main connective is 'or' or 'and', or are equivalent to such sentences, they can be symbolized as compound sentences of *SL*. But the question of when an English sentence is, or is equivalent to, a compound sentence that can be symbolized as a compound sentence of *SL* is often more complicated than the rather simple examples we have used would suggest. We have provided a table that gives rough English interpretations of the connectives of *SL*, but we need to be more precise. We begin with the concept of the **truth-functional use** of a sentential connective:

> A sentential connective of a formal or natural language is *used truth-functionally* if and only if it is used to generate a compound sentence

from one or more sentences in such a way that the truth-value of the generated compound is wholly determined by the truth-values of those one or more sentences from which the compound is generated, no matter what those truth-values may be.

English contains a number of sentential connectives that are always or nearly always used truth-functionally, some that are frequently used truth-functionally but also frequently used non-truth-functionally, and many that have no truth-functional uses. The connectives of *SL*, on the other hand, have only truth-functional uses. Because an understanding of how the connectives of *SL* work is required to appropriately symbolize English sentences in *SL* we here present the semantics or interpretation of the connectives of *SL*. The full semantics of *SL* is given in Chapter 3.

The following 'characteristic truth-tables' for the connectives of *SL fully define* the connectives *of SL*. It follows from these definitions that the connectives of *SL* have only truth-functional uses.

Negation

P	**~ P**
T	**F**
F	**T**

Conjunction

P	**Q**	**(P & Q)**
T	**T**	**T**
T	**F**	**F**
F	**T**	**F**
F	**F**	**F**

Disjunction

P	**Q**	**(P ∨ Q)**
T	**T**	**T**
T	**F**	**T**
F	**T**	**T**
F	**F**	**F**

Material Conditional

P	**Q**	**(P ⊃ Q)**
T	**T**	**T**
T	**F**	**F**
F	**T**	**T**
F	**F**	**T**

Material Biconditional

P	**Q**	**(P ≡ Q)**
T	**T**	**T**
T	**F**	**F**
F	**T**	**F**
F	**F**	**T**

The truth-value of a compound sentence of *SL* is fully determined by the truth-value(s) of its immediate component(s). Characteristic truth-tables display, in the columns to the left of the vertical line, all the combinations of truth-values the immediate components of compounds generated by the connective being defined can have. The truth-value the compound sentence has for each of those combinations of truth-values is displayed to the right of the vertical line.

NEGATIONS

The tilde is the connective of *SL* that roughly corresponds to the English connective 'It is not the case that'. The tilde turns sentences of *SL* that have the truth-value **T** into sentences that have the truth-value **F**—this is indicated by the first row of the table for negations—and it turns sentences of *SL* that have the truth-value **F** into sentences that have the truth-value **T**—this is indicated by the second row of the table. This means that a negation of *SL* is true if and only if the negated sentence is false.

CONJUNCTIONS

The ampersand is the connective of *SL* that roughly corresponds to the English connective 'and'. The characteristic truth-table for conjunctions has a '**T**' beneath the ampersand in the first row and only in the first row, and this is the only row in which there is a '**T**' beneath both '**P**' and '**Q**'. This means that a conjunction of *SL* is true if and only if both conjuncts are true and is false if and only if at least one conjunct is false.

DISJUNCTIONS

The wedge is the connective of *SL* that roughly corresponds to the English connective 'or'. The characteristic truth-table for disjunctions has a '**T**' under the wedge in every row in which there is a '**T**' beneath '**P**' *or* beneath '**Q**'. This means that a disjunction of *SL* is true if and only if at least one of its disjuncts is true and is false if and only if both of its disjuncts are false.

MATERIAL CONDITIONALS

The horseshoe is the connective of *SL* that roughly corresponds to the two-part English connective 'if . . . then'. The characteristic truth-table for material conditionals has a '**T**' under the horseshoe in every row in which there is a '**T**' under the consequent and in every row in which there is an '**F**' under the antecedent. This means that a material conditional of *SL* is true if and only if either the antecedent is false or the consequent is true. It is false if and only if the antecedent is true and the consequent is false.

MATERIAL BICONDITIONALS

The triple bar is the connective of *SL* that roughly corresponds to the English connective 'if and only if'. The characteristic truth-table for material biconditionals has a '**T**' under the triple bar in the row in which there is a '**T**' under both '**P**' and '**Q**' and in the row in which there is an '**F**' under both '**P**' and '**Q**'. This means that a material biconditional of *SL* is true if and only if its immediate components have the same truth-value and is false if and only if its immediate components have different truth-values.

We can now lay out the two-step process we will use in symbolizing English sentences in *SL*. The first step is to construct a truth-functional paraphrase of the sentence or sentences to be symbolized. Some examples will be useful. We have seen that sentences of English that can reasonably be symbolized as truth-functional compounds of *SL* are not themselves always compound sentences—for example, while 'The butler and the maid both hated Watson' is not a conjunction of two English sentences, it is equivalent to a conjunction of two English sentences. So the first step in symbolizing this sentence is to paraphrase it as

> The butler hated Watson <u>and</u> the maid hated Watson.

Our paraphrase is an explicit conjunction of two sentences. We have underlined the connective, in this case 'and', to indicate that it is being used purely truth-functionally. Here is another example:

> The pitcher for the home team will be either Betty or Margaret.

We paraphrase this sentence as an explicit disjunction of two sentences:

> Betty will be the pitcher for the home team <u>or</u> Margaret will be the pitcher for the home team,

again underlining the connective to indicate it is being used purely truth-functionally in the paraphrase.

We can paraphrase 'Margaret will be the pitcher for the home team if her shoulder has healed' as

> <u>If</u> Margaret's shoulder has healed <u>then</u> Margaret will be the pitcher for the home team.

In this case we made two changes to the original sentence. First, we reversed the order in which the component sentences occur, placing the sentence following 'if' first. Second, we replaced 'she' with 'Margaret' so that each sentence written alone is completely interpreted. That is, we have explicitly indicated to whom 'her' refers.

The purpose of paraphrasing an English sentence is to produce a sentence that can easily be symbolized in *SL*. When an English sentence is or can be paraphrased as a truth-functional compound, the paraphrase will obviously be a truth-functionally compound sentence and its structure will mirror the structure of the sentence of *SL* we will use to symbolize it. If an English sentence cannot be paraphrased as a truth-functionally compound sentence, then we will let that sentence serve as its own paraphrase and will symbolize it as an atomic sentence of *SL*.[5] Because paraphrases that are truth-functional compounds mirror the structure of the sentences that will symbolize them in *SL*, we will speak of them, as well as of sentences of *SL*, as being negations, conjunctions, disjunctions, material conditionals, and material biconditionals and as having main connectives and immediate components.

Symbolizing truth-functional paraphrases in *SL* is straightforward. If a paraphrase is not a truth-functional compound, we will symbolize the paraphrase as an atomic sentence of *SL*. If it is a truth-functional compound, then we symbolize it as a truth-functional compound of *SL* with the same structure.

Here is a group of sentences that can be paraphrased as negations and symbolized as sentences of the form ~ **P**. Note that only one of these English sentences contains the word 'not'.

The United States isn't a confederation of states.
Chlorine is a nonmetal.
Aristotle was unmarried.
Not everyone likes hip hop music.
Someone isn't telling the truth.
No one always tells the truth.

Paraphrasing the first four examples is straightforward:

<u>It is not the case that</u> the United States is a confederation of states.
<u>It is not the case that</u> chlorine is a metal.
<u>It is not the case that</u> Aristotle was married.
<u>It is not the case that</u> everyone likes hip hop music.

The fifth and sixth examples are less straightforward. It would be a mistake to paraphrase 'Someone isn't telling the truth' as

<u>It is not the case that</u> someone is telling the truth,

because this purported paraphrase is equivalent to 'No one is telling the truth', a far stronger claim than 'Someone isn't telling the truth'. A correct paraphrase for the fifth sentence is

<u>It is not the case that</u> everyone is telling the truth.

The sixth sentence, 'No one always tells the truth', is a denial of the claim made by the sentence 'There is someone who always tells the truth' and is therefore correctly paraphrased as

<u>It is not the case that</u> there is someone who always tells the truth.

[5]We refer to the sentences that result from the paraphrase process, including those that serve as their own paraphrases, as 'truth-functional paraphrases'. Note that those that serve as their own paraphrases are not truth-functionally compound sentences.

Using the symbolization key

U: The United States is a confederation of states.
C: Chlorine is a metal.
A: Aristotle was married.
E: Everyone likes hip hop music.
T: Everyone is telling the truth.
S: There is someone who always tells the truth.

we can symbolize the paraphrases as '~ U', '~ C', '~ A', '~ E', '~ T', and '~ S', respectively.

The following English sentences can be paraphrased as conjunctions:

Handel and Mozart both composed operas.
Beethoven composed symphonies and piano sonatas.
Beethoven composed nine symphonies, as did Mahler.
Mahler's *Kindertoten Lieder* are beautiful but also sad.

Here are our paraphrases:

Handel composed operas <u>and</u> Mozart composed operas.
Beethoven composed symphonies <u>and</u> Beethoven composed piano sonatas.
Beethoven composed nine symphonies <u>and</u> Mahler composed nine symphonies.
Mahler's *Kindertoten Lieder* are beautiful <u>and</u> Mahler's *Kindertoten Lieder* are sad.

The first three paraphrases clearly capture the full meaning of the sentences being paraphrased. But it is arguable that the fourth paraphrase does not capture the full meaning of the original, which uses the word 'but' rather than 'and' as a sentential connective. The word 'but' suggests a contrast or tension between a composition's being beautiful and its being sad, such that it is surprising to hear that a beautiful composition is at the same time sad. This suggestion is not present in the truth-functional paraphrase. Nonetheless, the paraphrase does capture what is asserted, rather than just suggested, by the original sentence. So what is asserted by the original sentence is true if both 'Mahler's *Kindertoten Lieder* are beautiful' and 'Mahler's *Kindertoten Lieder* are sad' are true.

Just how much of the content of the original sentence a truth-functional paraphrase must capture to be a reasonable paraphrase may depend on the context, but usually the loss of a suggestion will not matter to the logical analysis of a sentence or passage that has been truth-functionally paraphrased. Other English words that may be rendered as a truth-functional 'and' in paraphrases, some of which may also suggest an element of surprise or unexpectedness, include 'nevertheless', 'moreover', 'while', 'although', and 'albeit'.

Using the symbolization key

H: Handel composed operas.
M: Mozart composed operas.
S: Beethoven composed symphonies.
P: Beethoven composed piano concertos.
N: Beethoven composed nine symphonies.
A: Mahler composed nine symphonies.
B: Mahler's *Kindertoten Lieder* are beautiful.
K: Mahler's *Kindertoten Lieder* are sad.

our paraphrases can be symbolized as 'H & M', 'S & P', 'N & A', and 'B & K', respectively.

All of the following sentences can be paraphrased as disjunctions:

Maggie or Ronald will win the race.
Jim likes either jazz or hip hop.
Karen likes classical music, unless her tastes in music have changed.
At least one of the two finalists, Betty and Larry, will be very happy.

Appropriate paraphrases are

Maggie will win the race <u>or</u> Ronald will win the race.
Jim likes jazz <u>or</u> Jim likes hip hop.
Karen likes classical music <u>or</u> Karen's tastes in music have changed.
Betty will be very happy <u>or</u> Larry will be very happy.

The first two paraphrases are straightforward. The third sentence that we have paraphrased as a disjunction contains the sentential connective 'unless' rather than 'or', but it is clear that 'unless' in this sentence is correctly rendered as the truth-functional connective 'or'. If someone asks us what type of music Karen likes, and we know that Karen liked classical music the last time we talked with her, which was a year ago, we might say "Karen likes classical music, unless her tastes in music have changed". Here we mean that either she likes classical music (as she did a year ago) or her tastes have changed (in which case she might no longer like classical music). The fourth paraphrase may not jump out as an obvious truth-functional paraphrase if we only look at the original sentence, but it clearly captures what the original is saying.

We will use the following symbolization key to symbolize these paraphrases in *SL*:

M: Maggie will win the race
R: Ronald will win the race.
J: Jim likes jazz.
H: Jim likes hip hop.
K: Karen likes classical music.

C: Karen's tastes in music have changed.
B: Betty will be very happy.
L: Larry will be very happy.

Our symbolizations are 'M ∨ R', 'J ∨ H', 'K ∨ C', and 'B ∨ L', respectively.

Recall that disjunctions of *SL*, sentences of *SL* of the form **P** ∨ **Q**, are true if either **P** is true, or **Q** is true, or both are true. It is sometimes claimed that there are two uses of 'or' in English, one in which the 'or' means 'either this or that or both' and the other in which it means 'either this or that and not both'. The former is described as the inclusive use of 'or', the latter the exclusive use. It may appear that 'or' is being used in the exclusive sense in sentences such as

Louise Penny or Miles Blunt will win this year's Silver Dagger Award,

for surely two authors can't win the mystery writers' award in question. But we have made a misstep here. First, for all we know, the awards committee does sometimes award two or more authors the Silver Dagger Award in a single year. More importantly, if the rules governing the awarding of the Silver Dagger allow for only one winner per year, then *it is those rules*, not the meaning of 'or', that keep both Louise Penny and Miles Blunt from winning this year's award.

Those who think 'or' does have an exclusive use in English often cite the use of 'or' in choices we are offered. When a spokesperson for a state lottery announces that the grand prize winner can choose to receive $12 million in a lump sum or $1 million per year for the next fifteen years, we all know the winner won't be able to get both $12 million in a lump sum and $1 million a year for the next fifteen years. It is clearly one or the other and not both. And when studying a menu that contains the sentence 'With the Chef's Special you may have either an egg roll or hot and sour soup', almost everyone will know that they cannot get both an egg roll and hot and sour soup with the Chef's Special, at least not without paying extra. Whether we know these things because we recognize that 'or' is being used, in these cases of proffered choices, in the exclusive sense, or because we know about the customs and conventions prevalent when we are given a choice, is another matter.

If there is an exclusive sense of 'or' in English we can capture that sense in *SL*. For example, if we want to say that that Sally is in either New York or Chicago but not both, we can do so with the following paraphrase and symbolization:

(Sally is in New York <u>or</u> Sally is in Chicago) <u>and</u> <u>it is not the case that</u> (Sally is in New York <u>and</u> Sally is in Chicago)

(N ∨ C) & ~ (N & C)

Here we have used 'N' to symbolize 'Sally is in New York' and 'C' to symbolize 'Sally is in Chicago'.

The following four sentences can be paraphrased as material conditionals:

> If Sheila's hard work is recognized, she will get a raise.
> Pamela will get a raise if everyone does.
> Cynthia will get a raise provided she finishes her current assignment on time.
> Harry will get a raise only if his boss is a damn fool.

Appropriate paraphrases of our four examples are

> <u>If</u> Sheila's hard work is recognized <u>then</u> Sheila will get a raise.
> <u>If</u> everyone will get a raise <u>then</u> Pamela will get a raise.
> <u>If</u> Cynthia finishes her current assignment on time <u>then</u> Cynthia will get a raise.
> <u>If</u> Harry will get a raise <u>then</u> Harry's boss is a damn fool.

We will use the following symbolization key:

> S: Sheila's hard work is recognized.
> R: Sheila will get a raise.
> E: Someone will get a raise.
> P: Pamela will get a raise.
> F: Cynthia finishes her current assignment on time.
> C: Cynthia will get a raise.
> H: Harry will get a raise.
> B: Harry's boss is a damn fool.

The first paraphrase is straightforward and can be symbolized as '$S \supset R$'. Our paraphrase for the second sentence reverses the order of 'Pamela will get a raise' and 'Everyone will [get a raise]' in order to place the 'if' part of the sentence at the beginning of the paraphrase. This sentence is symbolized as '$E \supset P$'. The third example illustrates that not every English sentence that can be paraphrased as a material conditional contains the word 'if'. In this example, 'provided that' plays the role of 'if'. This paraphrase is symbolized as '$F \supset C$'. The sentences 'Cynthia will get a raise, assuming she finishes her current assignment on time' and 'Should Cynthia finish her current on time, she will get a raise' can both be paraphrased as '<u>If</u> Cynthia finishes her current assignment on time <u>then</u> Cynthia will get a raise'.

The fourth example is intended to illustrate the difference between 'if' and 'only if'. Note that the sentence being paraphrased is 'Harry will get a raise *only if* his boss is a damn fool', *not* 'Harry will get a raise *if* Harry's boss is a damn fool'. The latter tells us that if Harry's boss is a damn fool, then Harry will get a raise. But the former tells us that if Harry does get a raise, then his boss is a damn fool. These are not equivalent claims. Our symbolization is '$H \supset B$'.

To put the point more generally, when we are told that **p** only if **q** we are being told that if **p** is true **q** is true as well, and this is not the same as saying that if **q** is true **p** is also true. A university may require that all students complete two semesters of a foreign language before graduating. If so, then Kurt will graduate only if he has two semesters of a foreign language. But the university undoubtedly has other graduation requirements, for example, completing 120 semester hours of academic credit. If Kurt doesn't meet these additional requirements, he won't graduate—*even if* he does have two semesters of a foreign language.

English sentences of the form

p only if **q**

should therefore be paraphrased as sentences of the form

<u>if</u> **p** <u>then</u> **q**,

that is, the sentential component *following* 'if' becomes the *consequent*, not the antecedent, of the paraphrase.

On the other hand, sentences of the forms

if **p** (then) **q**
q if **p**
q provided that **p**
assuming **p**, **q**
q, assuming **p**

should all be paraphrased as sentences of the form

<u>if</u> **p** <u>then</u> **q**.

The following sentence can be straightforwardly paraphrased as a material biconditional:

> The global financial crisis will be resolved if but only if the world's major economic powers cut long-term spending.

Here is our paraphrase:

> The global financial crisis will be resolved <u>if and only if</u> the world's major economic powers cut long-term spending.

Note that in constructing this truth-functional paraphrase we replaced 'if but only if' with 'if and only if', and we did so for the reason that we earlier replaced the simple connective 'but' with 'and'. That is, the use of 'but' suggests, but does not assert, that it is not clear or obvious that the world's major economic powers will cut long-term spending. Our paraphrase can be symbolized as '$G \equiv W$', using the following symbolization key:

G: The global financial crisis will be resolved.
W: The world's major economic powers cut long-term spending.

Although sentences of English that are appropriately paraphrased as material biconditionals often contain the expression 'if and only if' or the variant 'if but only if', sentences containing the expression 'just in case' can also sometimes be paraphrased as material biconditionals. Consider the following sentences:

Fighter pilots carry parachutes just in case they have to eject from their planes.
The House will pass the tax reform bill just in case there is great public pressure for tax reform.

The first sentence clearly should *not* be paraphrased as 'Fighter pilots carry parachutes <u>if and only if</u> fighter pilots have to eject from their planes', for the paraphrase says that fighter pilots carry parachutes when they have to eject and only at such times. Clearly the English sentence allows for fighter pilots carrying parachutes at all times, whether or not these are times when they have to eject. The English sentence should therefore not be interpreted as a claim about when pilots carry parachutes, but rather as an explanation of why they carry parachutes, namely, to be prepared for emergencies. But the second sentence can correctly be paraphrased as a material biconditional:

The House will pass the tax reform bill <u>if and only if</u> there is great public pressure for tax reform.

This can be symbolized in *SL* as '$H \equiv G$', using the following symbolization key:

H: The House will pass the tax reform bill
G: There is great public pressure for reform.

2.2E EXERCISES

1. Paraphrase and then symbolize each of the following sentences, indicating which sentences the sentence letters you use symbolize.

a. Bob isn't a marathon runner.
*b. Albert and Bob are joggers.
c. If Carol is a jogger she is also a marathon runner.
*d. Some joggers are marathon runners.
e. Carol will run in the Boston marathon if and only if Albert does.
*f. Not all joggers are marathon runners.
g. Either Carol or Albert will run in the Boston marathon.
*h. If Carol will run in the Boston marathon so will Albert.

2. Paraphrase and then symbolize each of the following sentences, indicating which sentences the sentence letters you use symbolize.

a. If Felice vacations in Bermuda so will Clarence.

*b. Veronica will vacation in Bermuda only if Clarence does.

c. Veronica will vacation in Bermuda if Felice does.

*d. Either Clarence or Robert will vacation in Bermuda.

e. Veronica will vacation in Bermuda provided that Clarence will.

*f. Robert won't vacation in Bermuda.

2.3 MORE COMPLEX SYMBOLIZATIONS

In this section we will paraphrase and symbolize more complex sentences and sets of sentences in *SL*. Along the way, we will also illustrate and discuss some of the finer nuances of the symbolization process. We shall continue to symbolize English sentences in two stages, first constructing truth-functional paraphrases of the sentence or sentences to be symbolized and then symbolizing the paraphrases in *SL*. We begin by laying out guidelines for the construction of truth-functional paraphrases:

1. Determine whether the sentence to be symbolized can reasonably be paraphrased as a truth-functionally compound sentence.
   a. If it cannot, use the sentence as its own paraphrase.
   b. If it can, determine whether its immediate component(s) and their components can also reasonably be paraphrased as truth-functional compounds.
2. Use one or more of the connectives '<u>it is not the case that</u> . . .', '. . . <u>and</u> . . .', '. . . <u>or</u> . . .', '<u>if</u> . . . <u>then</u> . . .', and '. . . <u>if and only if</u> . . .' to construct truth-functional paraphrases of each sentence that can reasonably be paraphrased as a truth-functionally compound sentence.
3. Where applicable, use parentheses and square brackets to indicate which sentences are the immediate components of truth-functional compounds.
4. When paraphrasing an argument, present the paraphrased premises and conclusion in standard form. That is, list the paraphrased premises, draw a line beneath the last premise, and then list the paraphrased conclusion.
5. Reword the sentences being paraphrased so that all immediate components of the paraphrase are complete sentences with no cross-references between components, and if there are two or more wordings of the same claim, use just one wording in the paraphrase.

Some explanatory comments are in order. Many English sentences are *not* compound sentences. Among them are 'Canada is a member of the Commonwealth of Nations' and 'George W. Bush was the 43rd President of the United States'.

Guideline 1 specifies that such sentences should serve as their own paraphrases, as these are not compound sentences. Each such sentence will of course be symbolized by a single sentence letter of *SL*. There are also English sentences that contain sentences as proper constituents that should be used as their own paraphrases. One example is 'Archie believes that playing the lottery is the best way to get rich'. This sentence is formed by placing 'Archie believes that' in front of 'Playing the lottery is the best way to get rich'. But 'Archie believes that playing the lottery is the best way to get rich' cannot be paraphrased as a truth-functionally compound sentence in which 'Playing the lottery is the best way to get rich' is a component, because the truth-value of the former is not determined by the truth-value of the latter. Given only that a sentence is true, it does not follow that Archie believes it, and it does not follow that he does not believe it. Similarly, given only that a sentence is false it follows neither that Archie believes it nor that he does not believe it. Hence, 'Archie believes that playing the lottery is the best way to get rich' and all other sentences that cannot reasonably be paraphrased as truth-functional compounds should be used as their own paraphrases and symbolized as atomic sentences of *SL*. (Non-truth-functionally compound sentences will be further discussed in Section 2.4.)

Guideline 2 simply lists the connectives that are available for constructing truth-functionally compound paraphrases of English sentences. We shall prove in Chapter 6 that the structure of *every* truth-functionally compound sentence of English, no matter how complex, can be captured in a paraphrase that uses only the five truth-functional connectives listed in Guideline 2.

Guideline 3 calls for using parentheses and/or square brackets in paraphrases to indicate which sentences are being connected by which binary connectives. Doing so serves to eliminate ambiguities and also to mirror the syntax of *SL*, where parentheses are necessary to indicate grouping. Some English sentences that contain multiple sentential connectives are ambiguous, from a syntactic point of view. Consider

> Paul is taking saxophone lessons and Ellen is taking saxophone lessons or Karen is taking saxophone lessons.

Someone who asserts this sentence might intend to say that Paul is taking saxophone lessons and either Ellen or Karen is also taking saxophone lessons, making 'and' the main connective of the sentence. Alternatively, the intent might be to say that either Paul and Ellen are both taking saxophone lessons or Karen is taking saxophone lessons, making 'or' the main connective. That is, an English sentence of the form

> **p** and **q** or **r**

is syntactically ambiguous. From the syntax alone we don't know whether the intended meaning is

> **p** and (**q** or **r**)

or

(**p** and **q**) or **r**

In actual English discourse the context or the tone of voice or the emphasis a speaker places on one connective or the other often removes ambiguities like this. But *SL* does not contain ambiguities, and we shall reflect this in our truth-functional paraphrases by using parentheses to remove such ambiguities.

Some English sentences containing multiple sentential connectives are not ambiguous. These include sentences that can be recast as what we will call 'extended conjunctions' and 'extended disjunctions'. An example of the first sort is

Paul, Ellen, and Karen are all taking saxophone lessons.

This sentence can clearly be recast as:

Paul is taking saxophone lessons and Ellen is taking saxophone lessons and Karen is taking saxophone lessons.

A sentence that can be recast as an extended disjunction is

Either Paul or Ellen or Karen is taking saxophone lessons.

This sentence can be recast as

Paul is taking saxophone lessons or Ellen is taking saxophone lessons or Karen is taking saxophone lessons.

In neither case do we need to "figure out" which connective of the recast sentence is the main connective, as neither the original sentences nor their recastings are ambiguous. The first is true if and only if all three of the named individuals are taking saxophone lessons, the second if and only if at least one of them is taking saxophone lessons. However, because we will have occasion to symbolize such sentences in *SL*, where parentheses are required in sentences containing multiple ampersands or multiple wedges, we will use parentheses and/or square brackets to identify a main connective in paraphrasing such sentences. Of course there are multiple ways of doing this—that is, we can use parentheses to make the first occurrence of 'and' (or 'or'), or the second or the third, the main connective. All of these paraphrases are equally appropriate. For example, here are two equally acceptable truth-functional paraphrases of our original conjunction:

Paul is taking saxophone lessons <u>and</u> (Ellen is taking saxophone lessons <u>and</u> Karen is taking saxophone lessons), and

> (Paul is taking saxophone lessons <u>and</u> Ellen is taking saxophone lessons) <u>and</u> Karen is taking saxophone lessons.[6]

Guideline 4 is straightforward. But guideline 5 needs some explanation. One way to eliminate cross-reference is to replace pronouns with the terms for which they are going proxy. In paraphrasing

> If John is late he will have a good excuse,

it is obviously appropriate to replace 'he' with 'John'. More complex rewordings are often necessary. Suppose we are asked to paraphrase the following very simple argument:

> If Sally is late for class she will miss the discussion of Darwin's study of pigeons. Sally will be late for class, so she will miss the discussion.

A paraphrase of this argument that does *not* follow guideline 5 is

> <u>If</u> Sally is late for class <u>then</u> Sally will miss the discussion of Darwin's study of pigeons.
>
> Sally will be late for class.
>
> ---
>
> Sally will miss the discussion.

This paraphrase contains the following four distinct component sentences: 'Sally is late for class', 'Sally will miss the discussion of Darwin's study of pigeons', 'Sally will be late for class', and 'Sally will miss the discussion'. Using the four sentence letters 'S', 'M', 'W', and 'D', respectively, to symbolize these component sentences generates the following argument of *SL*:

$$S \supset M$$
$$W$$
$$\overline{}$$
$$D$$

This is not a valid argument of *SL*. Yet the original English argument is valid. The problem is that our paraphrase does not reflect the fact that in the original argument 'Sally is late for class' and 'Sally will be late for class' express the same claim. It is also implicit in the original argument that the discussion Sally

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[6]Since the grouping we use in extended conjunctions and extended disjunctions is arbitrary, it may appear that we should allow extended conjunctions and disjunctions in *SL* without the use of parentheses to indicate which connective is the main connective. We do not and cannot allow this because to apply the rules of the logical systems that we develop for *SL*, it will be necessary to know for every use of a binary connective which sentences are connected by that connective.

will miss is the discussion of Darwin's study of pigeons, not some other discussion. Hence the following rewording provides an appropriate paraphrase:

> <u>If</u> Sally will be late for class <u>then</u> Sally will miss the discussion of Darwin's study of pigeons.
>
> Sally will be late for class.
>
> ---
>
> Sally will miss the discussion of Darwin's study of pigeons.

Using 'S' to symbolize 'Sally will be late for class' and 'M' to symbolize 'Sally will miss the discussion of Darwin's study of pigeons' we can symbolize this paraphrased argument as follows:

$$\begin{array}{l} S \supset M \\ \underline{S} \\ M \end{array}$$

This argument *is* valid in *SL*.

The following argument calls for even more extensive rewording in the paraphrase:

> Either Jim will not pass the test or Jim spent last night studying logic. Jim's night was not spent poring over his logic text. Hence, Jim will fail the test.

Here is an *inappropriate* paraphrase of this argument, a paraphrase that ignores our fifth guideline:

> <u>It is not the case that</u> Jim will pass today's logic test <u>or</u> Jim spent last night studying logic.
>
> <u>It is not the case that</u> Jim's night was spent poring over his logic text.
>
> ---
>
> Jim will fail the test.

Symbolizing this paraphrase requires the use of four different sentence letters of *SL*:

$$\begin{array}{l} \sim J \vee S \\ \underline{\sim P} \\ F \end{array}$$

This is not a valid argument of *SL*. Yet, the English language argument with which we started is valid. Our paraphrase fails to reflect the fact that in this argument, 'Jim will not pass the test' and 'Jim will fail the test' are intended to be equivalent—these two sentences are making the same claim.[7] So too, 'Jim spent last night studying logic' is making the same claim as is 'Jim's night was spent poring over his logic text'.

A better paraphrase of our argument is

It is not the case that Jim will pass the logic test or Jim spent last night studying logic.

It is not the case that Jim spent last night studying logic.

It is not the case that Jim will pass the logic test.

This argument can be symbolized in *SL* using just two sentence letters, with 'J' symbolizing 'Jim will pass the logic test' and 'S' symbolizing 'Jim spent last night studying logic':

$\sim J \vee S$

$\sim S$

$\sim J$

And this argument will turn out to be a valid argument of *SL*.

Using our guidelines for constructing paraphrases we will now paraphrase and symbolize additional sentences, passages, and arguments in *SL*. Our first group of sentences concerns contemporary mystery writers. We will use the following symbolization key:

F: Ted has read *A Fine Red Rain*.
B: Ted has read *Bury Your Dead*.
D: Ted has read *The Old Fox Deceived*.
R: Ted has read *Rough Country*.

The first sentence we will paraphrase and symbolize is

Ted has read all of the books *A Fine Red Rain*, *Bury Your Dead*, *The Old Fox Deceived*, and *Rough Country*.

The paraphrase is straightforward:

(Ted has read *A Fine Red Rain* and Ted has read *Bury Your Dead*) and (Ted has read *The Old Fox Deceived* and Ted has read *Rough Country*),

[7]Failing and not passing are not always the same. If Sally is not enrolled in Jim's logic class, then she does not pass the test in that class, because she does not take it, but it is not true that she fails that test. More generally, that two claims can, in a given context, have a common paraphrase does not show that those two claims can be paraphrased as the same claim in every context.

as is the symbolization:

(F & B) & (D & R)

Because all of the connectives are ampersands, we could have grouped the conjuncts of both our paraphrase and our symbolization in several other ways, including:

Ted has read *A Fine Red Rain* <u>and</u> [Ted has read *Bury Your Dead* <u>and</u> (Ted has read *The Old Fox Deceived* <u>and</u> Ted has read *Rough Country*)]

F & [B & (D & R)]

Our next sentence can be paraphrased as an extended disjunction:

Ted has read at least one of the books *A Fine Red Rain, Bury Your Dead, The Old Fox Deceived,* and *Rough Country.*

An appropriate paraphrase is

(Ted has read *A Fine Red Rain* <u>or</u> Ted has read *Bury Your Dead*) <u>or</u> (Ted has read *The Old Fox Deceived* <u>or</u> Ted has read *Rough Country*).

(The grouping in this paraphrase is also arbitrary.) The symbolization is

(F ∨ B) ∨ (D ∨ R)

The sentence

Ted hasn't read any of the books *A Fine Red Rain, Bury Your Dead, The Old Fox Deceived,* or *Rough Country*

can be paraphrased as

(<u>It is not the case that</u> Ted has read *A Fine Red Rain* <u>and</u> <u>it is not the case that</u> Ted has read *Bury Your Dead*) <u>and</u> (<u>it is not the case that</u> Ted has read *The Old Fox Deceived* <u>and</u> <u>it is not the case that</u> Ted has read *Rough Country*)

and symbolized as

(~ F & ~ B) & (~ D & ~ R)

This sentence can also (and equivalently) be paraphrased as

<u>It is not the case that</u> [(Ted has read *A Fine Red Rain* <u>or</u> Ted has read *Bury Your Dead*) <u>or</u> (Ted has read *The Old Fox Deceived* <u>or</u> Ted has read *Rough Country*)]

and symbolized as

$$\sim [(F \vee B) \vee (D \vee R)]$$

The sentence

> Ted has read one but not both of the books *A Fine Red Rain* and *Bury Your Dead*

can be paraphrased as

> (Ted has read *A Fine Red Rain* or Ted has read *Bury Your Dead*) and it is not the case that (Ted has read *A Fine Red Rain* and Ted has read *Bury Your Dead*)

and symbolized as

$$(F \vee B) \ \& \sim (F \ \& \ B)$$

Note that ‘F ∨ B’ alone is not an acceptable symbolization of the sentence as the wedge of *SL* is inclusive, that is, the sentence ‘F ∨ B’ is true if Ted has read either or *both* of the books in question. Our current example can also be paraphrased and symbolized as follows:

> (Ted has read *A Fine Red Rain* and it is not the case that Ted has read *Bury Your Dead*) or (Ted has read *Bury Your Dead* and it is not the case that Ted has read *A Fine Red Rain*).

$$(F \ \& \sim B) \vee (B \ \& \sim F)$$

The sentence

> Ted has read exactly two of the books *A Fine Red Rain*, *Bury Your Dead*, and *The Old Fox Deceived*.

lists three books and says that Ted has read exactly two of them, but it doesn’t say which two. We can capture this claim by spelling out the three possibilities in our paraphrase:

> [(Ted has read *A Fine Red Rain* and Ted has read *Bury Your Dead*) and it is not the case that Ted has read *The Old Fox Deceived*] or ([(Ted has read *A Fine Red Rain* and Ted has read *The Old Fox Deceived*) and it is not the case that Ted has read *Bury Your Dead*] or [(Ted has read *Bury Your Dead* and Ted has read *The Old Fox Deceived*) and it is not the case that Ted has read *A Fine Red Rain*]).

This is symbolized as

$$[(F \,\&\, B) \,\&\, {\sim}\, D] \vee ([(F \,\&\, D) \,\&\, {\sim}\, B] \vee [(B \,\&\, D) \,\&\, {\sim}\, F])$$

Again, the grouping of the disjuncts is arbitrary, as is the grouping of the conjuncts within each disjunction. The sentence can also be paraphrased and symbolized as saying that Ted has read at least two of the books, but not all three:

> ([(Ted has read *A Fine Red Rain* and Ted has read *Bury Your Dead*) or (Ted has read *A Fine Red Rain* and Ted has read *The Old Fox Deceived*)] or (Ted has read *Bury Your Dead* and Ted has read *The Old Fox Deceived*)) and it is not the case that [(Ted has read *A Fine Red Rain* and Ted has read *Bury Your Dead*) and Ted has read *The Old Fox Deceived*].

$$([(F \,\&\, B) \vee (F \,\&\, D)] \vee (B \,\&\, D)) \,\&\, {\sim}\, [(F \,\&\, B) \,\&\, D]$$

We next paraphrase a series of sentences concerning various genres of music. We follow each paraphrase with a symbolization key and a symbolization of the paraphrase.

- Jazz is invigorating and classical music is uplifting, but neither is broadly popular.

This sentence can be paraphrased as a conjunction whose left conjunct is itself a conjunction and whose right conjunct is the negation of a disjunction:

> (Jazz is invigorating and classical music is uplifting) and it is not that case that (jazz is broadly popular or classical music is broadly popular).

J: Jazz is invigorating.
C: Classical music is uplifting.
B: Jazz is broadly popular.
P: Classical music is broadly popular.

$$(J \,\&\, C) \,\&\, {\sim}\, (B \vee P)$$

- Opera enthusiasts are small in number and very devoted to opera, but not always tolerant of other forms of music.

This sentence can also be paraphrased as a conjunction:

> Opera lovers are small in number and (opera lovers are very devoted to opera and it is not the case that opera lovers are always tolerant of other music).

O: Opera lovers are small in number.
D: Opera lovers are very devoted to opera.
T: Opera lovers are always tolerant of other forms of music.

O & (D & ~ T)

- Country and western music is wildly popular and is both funky and funny.

Our paraphrase is

Country and western music is wildly popular <u>and</u> (country and western music is funny <u>and</u> country and western music is funky).

C: Country and western music is wildly popular.
N: Country and western music is funny.
K: Country and western music is funky.

C & (N & K)

- Folk music was the rage in the 60s but has only a small following today, and it will make a comeback if and only if country and western music proves to be a fad, but it won't prove to be a fad.

A little reflection will show that our paraphrase should contain three conjunctions, one formed by the first 'but', a second by the 'and' occurring before 'it will make a comeback', and the third formed by the 'but' occurring before 'it won't prove to be a fad'. The paraphrase will also contain a material biconditional. We can treat the 'and' or either of the occurrences of 'but' as the main connective. We choose to treat the occurrence of 'and' as the main connective:

(Folk music was the rage in the 60s <u>and</u> folk music has only a small following today) <u>and</u> [(folk music will make a comeback <u>if and only if</u> country and western music proves to be a fad) <u>and</u> <u>it is not the case that</u> folk music will prove to be a fad].

R: Folk music was the rage in the 60s.
F: Folk music has only a small following today.
C: Folk music will make a comeback.
P: Country and western music proves to be a fad.

(R & F) & [(C ≡ P) & ~ P]

- Either hip hop is more popular than it deserves to be or there is more to it than there seems to be, but there isn't.

This sentence can be paraphrased as a conjunction whose left conjunct is a disjunction and whose right conjunct is a negation:

> (Hip hop is more popular than it deserves to be <u>or</u> there is more to hip hop than there seems to be) <u>and</u> <u>it is not the case that</u> there is more to hip hop than there seems to be.
>
> H: Hip hop is more popular than it deserves to be.
> M: There is more to hip hop than there seems to be.
>
> $(H \vee M)\ \&\ \sim M$

There are several points to note about the paraphrases and symbolizations we have just given. First, in paraphrasing our first and second sentences, we treated 'but' as surrogate for 'and'. Conversely, we point out that the word 'and' in the phrase 'country and western music' is not being used as a truth-functional connective. Our paraphrases and symbolizations also demonstrate that the mnemonic device of selecting a sentence letter based on an important word in the sentence to be symbolized is often of limited use. The paraphrase of our third sentence yielded three component sentences, all about country and western music:

> C: Country and western music is wildly popular.
> N: Country and western music is funny.
> K: Country and western music is funky.

We chose to use 'C' to symbolize the first of these, and 'C' may well remind us that it is symbolizing a sentence about country and western music, but it cannot remind us of which of the three component sentences about country and western music it symbolizes. We used 'N' and 'K' to symbolize the other two component sentences, and 'N' may serve to remind us that it symbolizes a sentence containing 'funny' (though we could equally well have used it to symbolize the third component sentence, which contains the word 'funky'). Similarly, our paraphrase of our fourth sentence yielded four component sentences, three of them about folk music. Again, the conclusion to be drawn from these examples is that the mnemonic device of using sentence letters that remind us of an important word in the sentence being symbolized is often of limited use.

We next paraphrase and symbolize several arguments. The first is

> Tim will go to the Blue Olive if and only if it is featuring a jazz trio but Susan will go if and only if the Blue Olive is featuring piano jazz. Ralph will go to the Blue Olive if Susan goes and Tim doesn't. Bill will go to the Blue Olive if they have a country and western band, but they don't. The Blue Olive is featuring piano jazz, not a jazz trio. So neither Tim nor Bill will go to the Blue Olive, but Susan and Ralph will.

Our paraphrase of this argument follows:

> (Tim will go to the Blue Olive if and only if the Blue Olive is featuring a jazz trio) and (Susan will go to the Blue Olive if and only if the Blue Olive is featuring piano jazz).
>
> If (Susan will go to the Blue Olive and it is not the case that Tim will go to the Blue Olive) then Ralph will go to the Blue Olive.
>
> (If the Blue Olive has a country and western band then Bill will go to the Blue Olive) and it is not the case that the Blue Olive has a country and western band.
>
> The Blue Olive is featuring piano jazz and it is not the case that the Blue Olive is featuring a jazz trio.
>
> ---
>
> It is not the case that (Tim will go to the Blue Olive or Bill will go to the Blue Olive) and (Susan will go to the Blue Olive and Ralph will go to the Blue Olive).

Using the symbolization key,

T: Tim will go to the Blue Olive.
J: The Blue Olive is featuring a jazz trio.
S: Susan will go to the Blue Olive.
P: The Blue Olive is featuring piano jazz.
R: Ralph will go to the Blue Olive.
C: The Blue Olive has a country and western band.
B: Bill will go to the Blue Olive.

we can symbolize the paraphrased argument as follows:

$$(T \equiv J) \ \& \ (S \equiv P)$$

$$(S \ \& \sim T) \supset R$$

$$(C \supset B) \ \& \sim C$$

$$P \ \& \sim J$$

$$\sim (T \vee B) \ \& \ (S \ \& \ R)$$

In subsequent chapters we will be able to show that this argument of *SL* is valid.

Our second argument is

> If the Outback Coral gets a liquor license before the end of the week it will feature a country and western band this weekend. Monica loves country and western music, and she will go to the Outback Coral this weekend if it does feature a country and western band. Eric hates country and western music but he is infatuated with Monica, and if Monica goes to the Outback Coral this weekend Eric

will also go, even though he will hate every minute of his time there. The Outback Coral will get a liquor license by the end of the week. Hence, Monica and Eric will both go to the Outback Coral this weekend and Eric will hate every minute of his time there.

We paraphrase the passage as an argument in standard form:

If the Outback Coral gets a liquor license before the end of the week then the Outback Coral will feature a country and western band this weekend.

Monica loves country and western music and (if the Outback Coral will feature a country and western band this weekend then Monica will go to the Outback Coral this weekend).

(Eric hates country and western music and Eric is infatuated with Monica) and (if Monica goes to the Outback Coral this weekend then (Eric will go to the Outback Coral this weekend and Eric will hate every minute of his time at the Outback Coral)).

The Outback Coral will get a liquor license by the end of the week.

Monica will go to the Outback Coral this weekend and (Eric will go to the Outback Coral this weekend and Eric will hate every minute of his time at the Outback Coral).

Our paraphrase of the argument yields eight sentences that will be symbolized as atomic sentences of *SL*. Our symbolization key follows. Note that we were not, in every case, able to use a sentence letter that bears a strong mnemonic connection to an important word in the sentence it symbolizes.

O: The Outback Coral will get a liquor license before the end of the week.
C: The Outback Coral will feature a country and western band this weekend.
L: Monica loves country and western music.
M: Monica will go to the Outback Coral this weekend.
H: Eric hates country and western music.
I: Eric is infatuated with Monica.
E: Eric will go to the Outback Coral this weekend.
T: Eric will hate every minute of his time at the Outback Coral.

$O \supset C$

$L \;\&\; (C \supset M)$

$(H \;\&\; I) \;\&\; [M \supset (E \;\&\; T)]$

O

$M \;\&\; (E \;\&\; T)$

In subsequent chapters we will also show that this argument of *SL* is valid.

Our third argument concerns contemporary mystery writers:

> At least one of the authors Louise Penny, Giles Blunt, Donna Leon, and Charles Todd will be nominated for this year's Gold Dagger Award. Everyone who is nominated will publish a new mystery this year. Neither Todd nor Blunt will publish a new mystery this year. Louise Penny will publish a new mystery this year if and only if Donna Leon does. Therefore, both Donna Leon and Louise Penny will publish new mysteries this year and at least one of them will be nominated for Gold Dagger Award.

Here is our paraphrase of this argument:

> (Louise Penny will be nominated for this year's Gold Dagger Award or Giles Blunt will be nominated for this year's Gold Dagger Award) or (Donna Leon will be nominated for this year's Gold Dagger Award or Charles Todd will be nominated for this year's Gold Dagger Award).
>
> [(If Louise Penny will be nominated for this year's Gold Dagger Award then Louise Penny will publish a new mystery this year) and (if Giles Blunt will be nominated for this year's Gold Dagger Award then Giles Blunt will publish a new mystery this year)] and [(if Donna Leon will be nominated for this year's Gold Dagger Award then Donna Leon will publish a new mystery this year) and (if Charles Todd will be nominated for this year's Gold Dagger Award then Charles Todd will publish a new mystery this year)].
>
> It is not the case that Charles Todd will publish a new mystery this year and it is not the case that Giles Blunt will publish a new mystery this year.
>
> Louise Penny will publish a new mystery this year if and only if Donna Leon will publish a new mystery this year.
>
> ---
>
> (Donna Leon will publish a new mystery this year and Louise Penny will publish a new mystery this year) and (Donna Leon will be nominated for this year's Gold Dagger Award or Louise Penny will be nominated for this year's Gold Dagger Award).

Note that we have paraphrased the first premise as an extended disjunction. The occurrence of 'and' in the first premise *does not* signal that the premise should be paraphrased as a conjunction. Rather it is used to specify the members of the group, *one of whom* will be nominated. The second premise, 'Everyone who is nominated will publish a new mystery this year' is about all potential nominees, not just the four authors named in the first premise. But the second premise is relevant to the validity of the argument only as it applies to the listed authors. Hence our paraphrase. We will use the following symbolization key:

P: Louise Penny will be nominated for this year's Gold Dagger Award.
B: Giles Blunt will be nominated for this year's Gold Dagger Award.
L: Donna Leon will be nominated for this year's Gold Dagger Award.
T: Charles Todd will be nominated for this year's Gold Dagger Award.
E: Louise Penny will publish a new mystery this year.
G: Giles Blunt will publish a new mystery this year.
D: Donna Leon will publish a new mystery this year.
C: Charles Todd will publish a new mystery this year.

(P ∨ B) ∨ (L ∨ T)

[(P ⊃ E) & (B ⊃ G)] & [(L ⊃ D) & (T ⊃ C)]

~ C & ~ G

E ≡ D

(D & E) & (L ∨ P)

There are, of course, multiple ways in which the first and second paraphrases and symbolizations can be grouped. On the other hand, the second premise *cannot* correctly be regrouped and symbolized as '[(P & B) & (L & T)] ⊃ [(E & G) & (D & C)]'. This sentence would serve as a symbolization of the claim that if they *all* win, then they will *all* publish a new mystery this year. It will turn out that the argument with the correct symbolization given above is a valid argument of *SL*.

Here's another argument:

> If Henry is after pure suspense he will read a Jeffrey Deaver mystery, and if he wants wonderfully rich characters and doesn't care about subtle plots, he will read a Martha Grimes mystery. But if he wants richly developed characters and a subtle plot he will read a Louise Penny mystery. Although Henry doesn't care about character development or subtle plots, he does want pure suspense, so Henry will read a Jeffrey Deaver mystery.

Here is our paraphrase:

(<u>If</u> Henry wants pure suspense <u>then</u> Henry will read a Jeffrey Deaver mystery) <u>and</u> [<u>if</u> (Henry wants well-developed characters <u>and</u> <u>it is not the case that</u> Henry cares about subtle plots) <u>then</u> Henry will read a Martha Grimes mystery].

<u>If</u> (Henry wants well-developed characters <u>and</u> Henry cares about subtle plots) <u>then</u> Henry will read a Louise Penny mystery.

It is not the case that (Henry wants well-developed characters or Henry cares about subtle plots) and Henry wants pure suspense.

Henry will read a Jeffrey Deaver mystery.

In paraphrasing the original argument we have taken 'Henry is after pure suspense' and 'Henry does want pure suspense' to express the same claim, and we have used the latter in our paraphrases. Henry's view about character development is also variously expressed in the original argument. The first premise mentions 'wonderfully rich characters', the second mentions 'richly developed characters', and the third simply mentions character development. We have paraphrased all three as 'Henry wants well-developed characters'. Here are our symbolization key and our symbolization of the argument:

S: Henry wants pure suspense.
D: Henry will read a Jeffrey Deaver mystery.
W: Henry wants well-developed characters.
P: Henry cares about subtle plots.
M: Henry will read a Martha Grimes novel.
L: Henry will read a Louise Penny mystery.

$(S \supset D) \ \& \ [(W \ \& \sim P) \supset M]$

$(W \ \& \ P) \supset L$

$\sim (W \vee P) \ \& \ S$

D

We will show in the next few chapters that this is also a valid argument of *SL*. Our final argument is

Christine will read a mystery if and only if there are no new science fiction novels in our library and there are no new science fiction movies available on Netflix. She will only read a mystery if it is set in the United States. Our library doesn't have any new science fiction novels and there are no new science fictions on Netflix. Donna Leon's mysteries are set in Venice, Louise Penny's mysteries and Giles Blunt's mysteries are set in Canada, and Charles Todd's mysteries are set in England. John Sandford's mysteries are set in the United States. Christine will therefore read a John Sandford mystery.

We paraphrase this as

Christine will read a mystery if and only if (it is not the case that there are new science fiction novels in our library and it is not the case that new science fiction movies are available on Netflix).

Christine will only read a mystery if it is set in the United States.

It is not the case that there are new science fiction novels in our library and it is not the case that new science fiction movies are available on Netflix.

(Donna Leon's mysteries are set in Venice and Louise Penny's mysteries are set in Canada) and (Giles Blunt's mysteries are set in Canada and Charles Todd's mysteries are set in England).

John Sandford's mysteries are set in the United States

Christine will read a John Sandford mystery.

We have used the second sentence of the original passage as its own paraphrase. This may seem strange, as the second sentence is an 'only if' claim and it may seem obvious that it should be paraphrased as

If Christine reads a mystery then it is set in the United States.

The problem is that in a truth-functional conditional what follows the 'then' must be an independent sentence that has a truth-value. 'It is set in the United States' is not such a sentence. Nor can it be turned into one by replacing 'it' with the name of a particular mystery, because 'it' does not refer to a particular mystery. If there were only a small number of mysteries, say two—*A Fine Red Rain* and *Rough Country*—then we could paraphrase the second premise as

(If Christine reads *A Fine Red Rain* then *A Fine Red Rain* is set in the United States) and (if Christine reads *Routh Country* then *Rough Country* is set in the United States).

But there are in fact an enormous number of mysteries, so it is not practical to paraphrase the second premise as a very long conjunction of material conditionals, each of which deals with one mystery. Rather, we will need the more powerful language *PL*, which is presented in Chapter 7, to adequately capture the structure of the second premise.

Here are our symbolization key and symbolizations:

C: Christine will read a mystery.
S: Our library has new science fiction novels.
N: New science fiction movies are available on Netflix.
U: Christine will only read a mystery if it is set in the United States.
D: Donna Leon's mysteries are set in Venice.
L: Louise Penny's mysteries are set in Canada.
G: Giles Blunt's mysteries are set in Canada.
T: Charles Todd's mysteries are set in England.

J: John Sandford's mysteries are set in the United States
R: Christine will read a John Sandford mystery.

C ≡ (~ S & ~ N)

U

(D & L) & (G & T)

J

R

This is not a valid argument of *SL*. Our paraphrase and symbolization do not bring out what is implicit in the original, that a mystery set in Venice is not set in the United States, that a mystery set in Canada is not set in the United States, and that a mystery set in England is not set in the United States. And even if these bits of geographic information were explicitly included in the argument and symbolization, the result would still not be a valid argument, because our symbolization of the second premise in *SL* does not show the relation between Christine's reading a mystery and the setting of that mystery, and also because John Sandford's mysteries are not the only mysteries set in the United States.

SUMMARY OF SOME COMMON CONNECTIVES

English Connective	*Paraphrase*	*Symbolization in SL*
not **p**	it is not the case that **p**	~ **P**
p and **q** **p** but **q** **p** however **q** **p** although **q** **p** nevertheless **q** **p** nonetheless **q** **p** moreover **q**	**p** and **q**	**P** & **Q**
p or **q** **p** unless **q**	**p** or **q**	**P** ∨ **Q**
p or **q** (exclusive sense)	**p** or **q** and it is not the case that (**p** and **q**)	(**P** ∨ **Q**) & ~ (**P** & **Q**)
if **p** then **q** **p** only if **q** **q** if **p** **q** provided that **p** **q** given **p**	if **p** then **q**	**P** ⊃ **Q**
p if and only if **q** **p** if but only if **q** **p** just in case **q**	**p** if and only if **q**	**P** ≡ **Q**

2.3E EXERCISES

1. Construct truth-functional paraphrases of the following sentences and symbolize those paraphrases in *SL*, using the following symbolization key:

P: The Red Sox improve their pitching.
G: The Red Sox have a good chance of winning the American League pennant.
Y: The Yankees will win the pennant.
F: The Red Sox falter.
T: The Twins win tonight.
M: The Mariners win tonight.
A: The Angels win tonight.
I: The Indians win tonight.
S: The Indians' starting pitcher can go the full nine innings.
N: The Angels move into first place.
H: The rain stops within an hour.
G: The game will be postponed.
R: The Royals are in the running for the pennant.

a. If the Red Sox improve their pitching they have a good chance of winning the American League pennant.
*b. The Yankees will win the pennant if the Red Sox falter and the Twins lose tonight.
c. If the Twins and the Mariners both lose tonight the Angels will move into first place.
*d. Assuming the rain stops within an hour the game will not be postponed.
e. The Indians will win tonight provided their starting pitcher can go the full nine innings.
*f. The Angels will move into first place only if the Twins and the Indians both lose tonight.
g. Assuming either the Twins or the Mariners win tonight, the Royals will be out of the running for the pennant.
*h. The Red Sox have a good chance of winning the pennant if and only if the Mariners and the Angels and the Twins all lose tonight.
i. The Royals are out of the race for the pennant and the Yankees will win the pennant if and only if the Twins win tonight and the Mariners and the Angels both lose tonight.
*j. The Red Sox have a good chance of winning the American League pennant but the Yankees will win the pennant if either the Red Sox falter or the Twins, the Angels, and the Mariners all win tonight.

2. Construct truth-functional paraphrases for the following, then provide a symbolization key and use it to symbolize your paraphrases in *SL*.
a. Either George or Emily will graduate with honors.
*b. Both George and Emily will graduate with honors or neither will.
c. At least one of George, Emily, Donna, and Fred will graduate with honors.
*d. If Donna graduates with honors so will Fred, and if Bob graduates with honors so will Emily.

e. Either they (Fred, George, Emily, and Donna) will all graduate with honors or none of them will.
*f. Either Fred won't graduate with honors or Emily and Donna both will.
g. Fred and George will graduate with honors if and only if Donna and Emily graduate with honors.
*h. Either George or Emily will graduate with honors but they won't both graduate with honors.
i. George won't graduate with honors but Fred will, and Donna will graduate with honors if and only if Emily does.
*j. If Emily and Donna don't both graduate with honors then neither George or Fred will graduate with honors.

3. Construct a truth-functional paraphrase of each of the following sentences, then provide a symbolization key and use it to symbolize your paraphrases.
a. If Felice vacations in Bermuda so will Clarence.
*b. Veronica will vacation in Bermuda only if both Clarence and Robert will also do so.
c. If either Felice or Veronica vacation in Bermuda they both will.
*d. Clarence will vacation in Bermuda only if Robert does and neither Felice nor Veronica do.
e. If Veronica vacations in Bermuda then Clarence will but Felice won't.
*f. Robert will vacation in Bermuda if and only if Clarence does, and Veronica will vacation in Bermuda if and only if Felice does.
g. Veronica will vacation in Bermuda if and only if Clarence doesn't, and Felice will vacation in Bermuda if and only if Robert does.
*h. Felice will vacation in Bermuda if and only if Veronica does and Robert doesn't, and Veronica will vacation in Bermuda if and only if Robert does and Clarence doesn't.

4. For each of the following, provide a truth-functional paraphrase and then symbolize your paraphrases in *SL*, indicating what sentence each of the sentence letters you use symbolizes.
a. *Casablanca*, *The Lion in Winter*, *Witness for the Prosecution*, *The Third Man*, and *Charade* will all be shown at this year's classical film festival.
*b. If Phil sees *Casablanca* he will enjoy Bogart's and Bergman's performances but he won't hear Bogart say "Play it again, Sam".
c. Phil will see *The Lion in Winter* only if Marion will and both of them will see *Charade*.
*d. Eric will see *The Lion in Winter* if and only if Betty does and if they see it they will love it.
e. If *Witness for the Prosecution* and *The Lion in Winter* are both screened at 8:00 pm, Marion and Phil will see *Witness for the Prosecution* and Eric and Betty will see *The Lion in Winter*.
*f. Phil will see *Charade* if and only if Audrey Hepburn and Cary Grant are both in it, and they are.
g. If it's the case that if Eric likes Katherine Hepburn then he'll see *The Lion in Winter*, then if Marion likes Eric she will see *The Lion in Winter*.
*h. If Claude Raines, Sydney Greenstreet, and Peter Lorre were in the movie Betty saw last night then she saw *Casablanca*.
i. Neither Betty nor Eric like James Coburn but they do both like Audrey Hepburn and if Audrey Hepburn is in *Charade* they will both see it (and she is).

5. Construct a truth-functional paraphrase of each of the following arguments, then provide a symbolization key and use it to symbolize your paraphrase of the argument in *SL*.

a. If Betty sees *Casablanca* and *The Third Man* then she won't see either *Witness for the Prosecution* or *The Lion in Winter*. She will see *Witness for the Prosecution* but she won't see *The Lion in Winter*. So either she won't see *Casablanca* or she won't see *The Third Man*.

*b. If Phil likes either Joseph Cotton or Orson Wells he will like *The Third Man*, if he sees it. If he likes either Peter O'Toole or Katharine Hepburn he'll like *The Lion in Winter*, if he sees it. He doesn't like either Joseph Cotton or Orson Wells, but he does like Katharine Hepburn. So if he sees *The Lion in Winter* he will like it.

c. Phil will see *The Third Man* if and only if he likes both Joseph Cotton and Orson Wells, and he will see *Witness for the Prosecution* if and only if he likes both Marlene Dietrich and Charles Laughton. He doesn't like either Joseph Cotton or Orson Wells, but he does like Marlene Dietrich and Charles Laughton. So Phil will see *Witness for the Prosecution*.

*d. Betty will see either *The Lion in Winter* or *Witness for the Prosecution*. Marion will see *Casablanca* and *Charade*. If Betty sees *Witness for the Prosecution* Eric won't, but he will see *Casablanca* if Marion does. Betty won't see *Witness for the Prosecution*, and she will see *The Lion in Winter* if and only if Phil does. So Phil will see *The Lion in Winter*.

6. Construct truth-functional paraphrases of each of the following passages. If a passage is an argument, present your paraphrase of the argument in standard form. Provide symbolization keys for your paraphrases of these passages and symbolize your paraphrases in *SL*.

a. Fred will go to New York only if he can get a first class air ticket and get tickets to a Yankees game. Fred will go to Chicago only if he can travel by train and get tickets to a White Sox game. He can't get a first class air ticket and he can't get tickets to a White Sox game, so he won't go to either New York or Chicago.

*b. If Lisa goes on vacation it will be to either Toronto, Montreal, Quebec, or Vancouver. If she goes to Toronto she will visit the University of Toronto; if she goes to Montreal she will eat great French food; if she goes to Quebec she will visit the Plains of Abraham; and if she goes to Vancouver she will go whale watching. She won't visit the University of Toronto; she won't eat great French food; and she won't go whale watching. So if she goes on vacation she will visit the Plains of Abraham.

c. Alice will go to Vienna if but only if Burt is willing to go with her and Burt speaks German. If Alice does go to Vienna she will take the Orient Express to Istanbul, unless Burt refuses to travel by train. Burt is willing to go with Alice to Vienna and he does speak German, but he won't travel by train. Hence if Alice goes to Vienna she will not take the Orient Express to Istanbul.

*d. Ben will go either to Duluth or to Kansas City. If it is the case that when Ben travels he travels by train, then if he travels to Duluth there is a train to Duluth and if he travels to Kansas City there is a train to Kansas City. Ben travels only by train and there is no train to Duluth. So Ben will travel to Kansas City and there is a train to Kansas City.

e. Charles Todd's mysteries are good mysteries. A good mystery has memorable characters, a plot that keeps the reader in suspense, and contains enough factual information to allow the reader to actually learn some interesting things; and Charles Todd's mysteries have all of these features.

2.4 NON-TRUTH-FUNCTIONAL USES OF CONNECTIVES

In Section 2.2 we introduced the notion of the truth-functional use of sentential connectives. As we explained there,

> A sentential connective, of a formal or a natural language, is used *truth-functionally* if and only if it is used to generate a compound sentence from one or more sentences in such a way that the truth-value of the generated compound is wholly determined by the truth-values of those one or more sentences from which the compound is generated, no matter what those truth-values may be.

The sentential connectives of *SL* are fully defined by their characteristic truth-tables and therefore have only truth-functional uses. As we have seen, we can often paraphrase English compounds as truth-functional compounds without weakening or distorting the content of the sentences being paraphrased. Paraphrases that are negations, conjunctions, and disjunctions often capture all or almost all of the content of the sentences being paraphrased. A sentence such as 'Aristotle and Alexander were both Greek' can be paraphrased as

> Aristotle was Greek <u>and</u> Alexander was Greek

with no loss of content. The English sentence says neither more nor less than the truth-functional paraphrase. In contrast, English conditionals frequently express links or connections between their antecedents and consequents that are lost when we paraphrase them as material conditionals. For example, consider the sentence

> Assuming the rain stops within an hour, the game will not be postponed.

This sentence is appropriately paraphrased as

> <u>If</u> the rain stops within an hour <u>then</u> <u>it is not the case that</u> the game will be postponed

and the paraphrase can be symbolized as '$S \supset \sim P$'. But it can be argued that our paraphrase does not capture all of the content of the sentence it paraphrases. In the original there is at least the suggestion that the rain's stopping, if it does, will be the reason the game will not be postponed and that the rain's not stopping, if it doesn't, will be the reason for the game's being postponed. Often this kind of loss of content will not matter for the purposes at hand, for example, determining the validity of an argument. But when an English conditional is based on a scientific law, paraphrasing that conditional as a material conditional can be problematic. An example is

> If this rod is made of metal, then it will expand when heated.

A simple law of physics lies behind this claim: all metals expand when heated, and the conditional is in effect claiming that if the rod in question is made of metal then heating it will *cause* it to expand. A paraphrase of this causal claim as a material conditional does not capture this causal connection. The failure to capture such causal connections may or may not be acceptable, depending on the context and on what questions we are asking about the sentence or set of sentences being paraphrased.

When we use the tools that we develop in subsequent chapters to analyze sentences and sets of sentences of *SL,* the results that we obtain will apply directly to the sentences and sets of sentences of *SL* we are analyzing and to the *truth-functional paraphrases* those sentences symbolize. The results will also apply to the English sentences from which the paraphrases are obtained to the extent, and only to the extent, that the paraphrases capture the content of the original sentences. Here's an example of how things can go wrong if we ignore this caveat. Suppose some benighted person incorrectly believes that metals contract when they are heated. Such a person might make the following claim about a rod whose composition is unknown: 'If this rod is made of metal, then this rod will contract when heated'. Taken as a causal claim, this is clearly false. Metals expand when heated; they don't contract. Here is a truth-functional paraphrase of the claim, and a symbolization of that paraphrase in *SL*:

<u>If</u> this rod is made of metal <u>then</u> this rod will contract when heated.

$M \supset C$

We have used 'M' to symbolize 'This rod is made of metal' and 'C' to symbolize 'This rod will contract when heated'. Now suppose that the rod is in fact plastic, not metal. Then the antecedent of '$M \supset C$' and of the paraphrase it symbolizes are both false, making the material conditional of *SL* and our paraphrase *both true*, even though the English sentence we paraphrased and symbolized is clearly false. In this case, where the alleged causal connection between antecedent and consequent is crucial to the claim being made, it is wise to treat the original sentence as a non-truth-functional compound and symbolize it as an atomic sentence of *SL*.

Of course, there are many English conditionals that can be appropriately paraphrased as material conditionals with no loss of content. We are all familiar with, and probably have made, claims of the sort

If such-and-such then I'm a such-and-such,

where the first 'such-and-such' is replaced by some very improbable claim and the second with a known falsehood. For example, if someone tells us that Jones, whom we know to be barely literate, is going to write the great American novel, one of us might comment 'If Jones can write the great American novel I can leap tall buildings in a single bound'. We all know the consequent of this conditional is false. By asserting the conditional, knowing the consequent is false, the speaker is implicitly asserting that the antecedent is also false, and the antecedent's being false makes the conditional true.

There are other English conditionals called 'subjective conditionals' that cannot adequately be paraphrased as material conditionals. Here are two examples:

> If Harry were to win the lottery, he would give all the proceeds to charity.

and

> If Hitler had not invaded Russia, he would have defeated Great Britain and won the Second World War.

We might be tempted to paraphrase the claim about Harry as a material conditional, that is, as

> <u>If</u> Harry wins the lottery <u>then</u> Harry will give all the proceeds to charity.

A material conditional is true when its antecedent is false. Now suppose that the antecedent of our paraphrase is false; Harry does not win the lottery (as will almost certainly be the case). Our paraphrase is then true. But Harry's failure to win hardly makes the original subjunctive claim true. Suppose we know that Harry is by nature not a generous person, and we know that he has never given a dime to charity in his life. Moreover he has frequently railed against those who do give to charity. If we know all of this, then we will reject the subjunctive conditional. Harry is just not the sort of person who gives money to charity. So we will conclude that it is not the case that if Harry were to win the lottery he would give all the proceeds to charity. That he did not win the lottery is irrelevant to this reasoning.

Our second example of a subjunctive conditional also cannot be paraphrased as a material conditional. All historians know that Hitler *did* invade Russia and *did not* win the Second World War. But they do not take those facts to determine the truth-value of the above subjunctive conditional concerning Hitler. In fact, historians continue to disagree about the truth-value of that subjunctive conditional.

English has a large number of non-truth-functional connectives. 'I believe that . . .' is one. Attach 'I believe that' to any sentence of English that has a truth-value and the result is a sentence of English that has a truth-value. But the result is not a truth-functional compound. Given any sentence of the form 'I believe that **p**', the truth-value of that sentence is not determined by the truth-value of **p**. No matter how obviously false **p** may be, I might still believe it, and no matter how obviously true it may be, I may still not believe it. 'It is alleged that' is not a truth-functional connective for similar reasons. Attaching 'It is alleged that' to any sentence with a truth-value yields a sentence with a truth-value, but the truth-value of the sentence to which 'It is alleged that' is attached does not determine the truth-value of the resulting sentence. Suppose that it is false that Senator Bigmouth took a bribe when he was mayor of Littletown. ‘It is alleged that Senator Bigmouth took a bribe when he was mayor of Littletown’ may nonetheless be true. All sorts of false things are alleged. And all sorts of true things are not alleged. Similar to the case of ‘I believe that’, the truth-value of ‘Senator Bigmouth took a bribe when he was mayor of Littletown’ and the truth-value of ‘It is alleged that Senator Bigmouth took a bribe when he was mayor of Littletown’ are logically independent. They can both be true, they can both be false, the first can be true while the second is false, and the second can be true while the first is false. Other non-truth-functional unary connectives of English include

> It is probable that
> Necessarily
> It would not be surprising if
> We are convinced that
> We hope that . . .
> I know that . . .

The truth-value of a sentence formed by attaching any one of these expressions other than ‘I know that . . .’ to a sentence **p** that has a truth-value is logically independent of the truth-value of **p**. The generated compound and **p** may both be true, they may both be false, **p** may be true and the compound false, and **p** may be false and the compound true. ‘I know that’ is different in that if this connective is attached to a false sentence then the compound that is generated is also false. But the compound may be either true or false when the sentence to which ‘I know that’ is attached is true.

Though connectives of the sort we have been discussing are all non-truth-functional, some of the compound sentences they generate can be paraphrased as truth-functional compounds. An example is

> Commentators believe the Republicans will retain control of the House and the Democrats will retain control of the Senate.

This claim can reasonably be paraphrased as the truth-functionally compound sentence

> Commentators believe the Republicans will retain control of the House <u>and</u> commentators believe the Democrats will retain control of the Senate.

This paraphrase is a truth-functional compound, a conjunction, each of whose conjuncts is a non-truth-functional compound. But we must be careful here.

> Commentators believe the Republicans will gain control of the Senate or of the House

is not reasonably paraphrased as

> Commentators believe the Republicans will gain control of the Senate <u>or</u> commentators believe the Republicans will gain control of the House.

Commentators may believe the Republicans will gain control of at least one chamber, the House or the Senate, but have no opinion about which chamber it will be. As another example, consider the flipping of a fair coin. We all believe that the coin will either come up heads or come up tails. But this is not equivalent to

> We all believe the coin will come up heads <u>or</u> we all believe the coin will come up tails.

There are also binary connectives of English that are never used truth-functionally. One is 'because'. Consider the sentence

> Henry will not read *Drawing Conclusions* because it is set in Venice.

The truth-value of this compound is *not* wholly determined by the truth-values of its immediate components. While the falsity of either 'Henry will not read *Drawing Conclusions*' or '*Drawing Conclusions* is set in Venice' is sufficient for the falsity of the compound, the truth of these components does not determine the truth-value of the compound. It is true that *Drawing Conclusions* is set in Venice and it may be true that Harry will not read it, but the reason he will not read it may have nothing to do with its being set in Venice. Perhaps the reason is that Henry's library doesn't have a copy and Henry is too cheap to buy a copy.

The connective 'before' is also a non-truth-functional connective. Consider the sentences

> Jimmy Carter was elected president before Ronald Reagan was elected president

and

> Ronald Reagan was elected president before Jimmy Carter was elected president.

The component sentences 'Ronald Reagan was elected president' and 'Jimmy Carter was elected president' are both true, but the first compound sentence is true while the second is false. Hence the truth-values of the components do not, in every case, determine the truth-value of the compound sentences and 'before' is, therefore, not a truth-functional connective. The same is true of 'after'. More generally, given that either **p** is false or **q** is false, we may conclude that both

> **p** before **q**

and

p after **q**

are false, but we cannot conclude anything about the truth or falsity of either of these claims *given only that* both **p** and **q** are true.

The safest policy for paraphrasing non-truth-functionally compound sentences is to let them be their own paraphrases and symbolize them as atomic sentences of *SL*. However, there are cases in which we can construct truth-functionally compound sentences of English that capture some of the content of non-truth-functionally compound English sentences, and it is sometimes useful to do so. First, a definition. We have noted several times that a paraphrase or proposed paraphrase fails to capture all of the content of the sentence being paraphrased. In such cases the paraphrase is "weaker" than the original. What it means to say that one sentence is weaker than, or stronger than, another is not entirely clear in ordinary English, and we therefore provide a **stipulative** definition of these terms that states what logicians mean when we use the words 'weaker' and 'stronger' to describe relationships between sentences:

> A sentence **p** of a natural or formal language is *stronger* than a sentence **q** of a natural or formal language (and **q** is *weaker* than **p**) if and only if **q** follows from **p** but **p** does not follow from **q**.[8]

For example, 'Aristotle was Greek and Alexander was Greek' is stronger than 'Aristotle was Greek', because the latter follows from the former, but not vice versa. For the same reason, 'Aristotle was Greek' is weaker than 'Aristotle was Greek and Alexander was Greek'.

Here is an argument all of whose premises are conditionals:

> If the rain continues the game will be postponed. If the game is postponed Bronson won't have to pitch today. If Bronson won't have to pitch today he will be ready to pitch tomorrow. If Bronson is ready to pitch tomorrow his team will win tomorrow. The rain will continue. So Bronson's team will win tomorrow.

This argument, which is valid, connects a series of envisioned events, the first being its continuing to rain today and the last being Bronson's team's winning tomorrow. Arguably, the envisioned events are presented as being connected in more than a truth-functional way. For example, it is at least implicit that if the rain continues, its doing so will cause the game to be postponed, and that if the game is postponed, the postponement will be responsible for Bronson's not having to pitch today, and that if Bronson's not having to pitch today will ensure that he'll be ready to pitch tomorrow, and that his being ready to pitch

[8]This is a stipulative definition because it does not fully accord with all the ways 'stronger than' and 'weaker than' are used in ordinary English. For example, most of us would take 'There is a cougar in the yard' to be a stronger claim than is 'I think there is a cougar in the yard'. But 'There is a cougar in the yard' does not follow from 'I think there is a cougar in the yard'. Such uses of 'stronger' in ordinary English perhaps convey that one claim conveys more reliable, or more important information than does another.

tomorrow will lead to and be responsible for his team's victory. These implicit causal relationships are lost in the following truth-functional paraphrase of the argument:

> If the rain will continue then the game will be postponed.
>
> If the game will be postponed then it is not the case that Bronson will have to pitch today.
>
> If it is not the case that Bronson will have to pitch today then Bronson will be ready to pitch tomorrow.
>
> If Bronson will be ready to pitch tomorrow then Bronson's team will win tomorrow.
>
> The rain will continue.
>
> ---
>
> Bronson's team will win tomorrow.

But the conclusion of the paraphrase is identical to the conclusion of the original, and it does follow from the paraphrased premises. And since each of the paraphrased premises is weaker than (and therefore follows from) the premise it paraphrases, the conclusion also follows from the original premises, and so we may conclude that the original argument is valid.

In paraphrasing the original argument we weakened each premise, by replacing a causal conditional with a material conditional. Causal conditionals are stronger than material conditionals. For example, it follows from the causal conditional

> If this rod is made of metal it will expand when heated,

that either the rod is not made of metal or it will expand when heated, which is all that the material conditional

> If this rod is made of metal then it will expand when heated

comes to, but the causal conditional does not follow from the material conditional.

We have seen that if we weaken the premises of an argument in the paraphrase process but do not weaken the conclusion, and the paraphrase and its symbolization turn out to be valid, we may safely conclude that the original argument is also valid. *But* if the paraphrased argument and its symbolization turn out to be *invalid* we *cannot* conclude that the original argument is invalid. That a conclusion does not follow from one set of premises (our paraphrases of the original premises) does not show that it does not follow from a stronger set of premises (the premises of the original argument). Hence while we can sometimes use a paraphrase whose premises are weaker than the premises of the original argument to show that the original argument is valid, we can never use such paraphrases to show the argument being paraphrased is invalid.

Here is a simple example that illustrates this point.

> Aristotle and Plato were both Greek. If Aristotle was Greek he wasn't Roman, and if Plato was Greek he wasn't Roman. So neither Aristotle nor Plato was Roman.

This is obviously a valid argument, as are its truth-functional paraphrase and symbolization:

Aristotle was Greek and Plato was Greek.

(If Aristotle was Greek then it is not the case that Aristotle was Roman) and (if Plato was Greek then it is not the case that Plato was Roman).

It is not the case that Aristotle was Roman and it is not the case that Plato was Roman.

Using 'A' to symbolize 'Aristotle was Greek', 'P' to symbolize 'Plato was Greek', 'R' to symbolize 'Aristotle was Roman', and 'L' to symbolize 'Plato was Roman', we can symbolize our paraphrased argument as

$$\begin{array}{l} A \;\&\; P \\ \underline{(A \supset \sim R) \;\&\; (P \supset \sim L)} \\ \sim R \;\&\; \sim L \end{array}$$

This symbolic argument is valid. Now suppose we weaken our paraphrase of the first premise by replacing it with 'Aristotle was Greek', a sentence that is clearly weaker than (because if follows from) the first premise of the original argument. The symbolization of our revised paraphrase will be

$$\begin{array}{l} A \\ \underline{(A \supset \sim R) \;\&\; (P \supset \sim O)} \\ \sim R \;\&\; \sim O \end{array}$$

This symbolic argument is invalid, as is the truth-functional paraphrase it symbolizes. But the original argument is valid. Again, showing that a paraphrased argument is invalid where the premises of the paraphrase are weaker than the premises of the original argument *does not show* that the original argument is invalid.

Here is a more interesting case in which weakening the premises of an argument in paraphrasing it is both appropriate and useful. Suppose that a detective reasons as follows:

> If Williams is the murderer he had to be in Philadelphia on the 5th. Because we know that Williams was in Rome on the 5th, we know that he was not in Philadelphia on the 5th. So Williams isn't the murderer.

This seems to be a valid piece of reasoning, and if we use 'Williams was in Rome on the 5^{th} and it is not the case that Williams was in Philadelphia on the night of the 5^{th}, for the second premise, it seems that we can capture the structure of that reasoning:

> If Williams is the murderer then Williams was in Philadelphia on the 5^{th}.
>
> Williams was in Rome on the 5^{th} and it is not the case that Williams was in Philadelphia on the 5^{th}.
>
> ---
>
> It is not the case that Williams is the murderer.

Our paraphrase of the first premise is weaker than the premise it paraphrases: we have replaced 'had to be in Philadelphia' with 'was in Philadelphia'. Our paraphrase of the second premise is weaker than the original and follows from it. Both **p** and **q** follow from causal claims of the sort

> Because **p**, **q**

and **p** follows from

> We know that **p**,

though not vice versa. So

> We know that Williams was in Rome on the 5^{th} and we know that Williams was not in Philadelphia on the 5^{th}

follows from the original second premise. And our paraphrase follows from this conjunction of two knowledge claims. Using obvious choices of sentence letters, we can symbolize the paraphrase as

$$\begin{array}{l} W \supset P \\ R \ \& \sim P \\ \hline \sim W \end{array}$$

This is a valid argument of *SL*.

Similarly, although English subjunctive conditionals are not truth-functional compounds, it is sometimes possible and appropriate to use material conditionals as paraphrases of subjunctive conditionals. Suppose that a doctor who is testifying at an inquest argues as follows:

> Had the deceased died of strychnine poisoning, there would have been traces of that poison in the body. The autopsy would have found those traces had they been there. The autopsy did not reveal any traces of strychnine. Hence the deceased did not die of strychnine poisoning.

Replacing the subjunctive conditionals with material conditionals we obtain the following paraphrase of this argument:

> If the deceased died of strychnine poisoning then there were traces of strychnine in the body.
>
> If there were traces of strychnine in the body then the autopsy found traces of strychnine in the body.
>
> It is not the case that the autopsy found traces of strychnine in the body.
>
> ---
>
> It is not the case that the deceased died of strychnine poisoning.

Using obvious choices of sentence letters, we can symbolize this argument in *SL* as

$$
\begin{array}{l}
S \supset T \\
T \supset F \\
\sim F \\
\hline
\sim S
\end{array}
$$

This is a valid argument of *SL*, as we will be able to show in subsequent chapters.

We have discussed when it is appropriate to weaken the premises of an argument when paraphrasing them. Recall that entailment is a notion that nearly parallels that of validity (the difference being that some sentences are entailed by the empty set but there are no arguments with no premises). Accordingly, it is sometimes appropriate to weaken the members of a set when trying to determine whether that set entails a given sentence.

Care must also be taken when weakening or strengthening a sentence in the paraphrase process when we are concerned with determining the consistency of a set of sentences, the equivalence of sentences, or the logical status of a sentence (logically true, logically false, or logically indeterminate). For example, if we are interested in whether a set of sentences of English is consistent and in the paraphrase process we weaken one of the members of the set, then showing that the set consisting of the paraphrased sentences is consistent will not establish that the original set of sentences is consistent. And if in the paraphrase process we strengthen one of the members of the set, then showing that the set consisting of the paraphrased sentences is inconsistent will not show that the original set is inconsistent, though if the set of paraphrased sentences turns out to be consistent, so is the original set. Similarly, if we are interested in whether two sentences are equivalent and weaken or strengthen either or both of the sentences in the paraphrase process, then showing that the paraphrases are, or are not, equivalent will not, in general, constitute showing that the original sentences are or are not equivalent.

What can be said, and we have so said before, is that the results we obtain by using the techniques developed in subsequent chapters to test for validity, entailment, consistency, and the other core semantical concepts apply directly only to the paraphrases and symbolizations we have constructed.

2.4E EXERCISES

1. Paraphrase and symbolize each of the following sentences that can reasonably be paraphrased as a truth-functional compound. If a sentence cannot be so paraphrased, explain why this is so. Provide a symbolization key when it is not obvious what sentence your sentence letters are symbolizing.

a. It's likely that either the Boston Red Sox or the New York Yankees will win the World Series this year.
*b. Either Rocky or George knows what time the concert starts.
c. Marcie thinks that either Helen or Stephanie will be elected.
*d. Tamara won't be visiting tonight because she is working late.
e. Although Tamara won't stop by, she has promised to phone early in the evening.
*f. If the victim had been strangled there would have been marks on his throat, and there weren't.
g. John believes that our manuscript has been either lost or stolen.
*h. John believes that our manuscript has been stolen, and Howard believes that it has been lost.
i. The defendant confessed only after much of her testimony was discredited.
*j. It is possible that the Twins will win tonight and possible that the Red Sox will win tonight, but it is not likely that they will both win tonight.

2. Construct truth-functional paraphrases of the premises and conclusions of the following arguments, provide symbolization keys, and symbolize your paraphrases in *SL*.

a. The murder was committed by the maid only if she believed her life was in danger. Had the butler done it, it would have been done silently and the body would not have been mutilated. As a matter of fact it was done silently; however, the maid's life was not in danger. The butler did it if and only if the maid failed to do it. Hence the maid did it.
*b. If this piece of metal is gold, then it has atomic number 79. Nordvik believes this piece of metal is gold. Therefore Nordvik believes this piece of metal has atomic number 79.
c. If Charles Babbage had had the theory of the modern computer and had had modern electronic parts, then the modern computer would have been developed before the beginning of the twentieth century. In fact, although he lived in the early nineteenth century, Babbage had the theory of the modern computer. But he did not have access to modern electronic parts, and he was forced to construct his computers out of mechanical gears and levers. Therefore, if Charles Babbage had had modern electronic parts available to him, the modern computer would have been developed before the beginning of the twentieth century.

GLOSSARY

TRUTH-FUNCTIONAL USE OF A CONNECTIVE: A sentential connective, of a formal or a natural language, is used *truth-functionally* if and only if it is used to generate a compound sentence from one or more sentences in such a way that the truth-value of the generated compound is wholly determined by the truth-values of those one or more sentences from which the compound is generated, no matter what those truth-values may be.