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*Edited by*  
MARY LOUISE GILL and  
PIERRE PELLEGRIN

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Ancient Philosophy

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## Stoic Logic

KATERINA IERODIAKONOU

## Introduction

*Logic as a part of philosophy*

According to most Stoic philosophers, logic is a part of philosophy, the other parts being physics and ethics (Aët. 1, prooem. 2 = LS 26A; D.L. 7.39 = LS 26B1). The Stoics distinguish these three parts of philosophy, because each part has its own particular subject-matter and aim, yet they are at the same time inseparably intertwined. But what is the subject-matter of Stoic logic? The Stoics do not use the term “logic” (*logikē*) as we do nowadays. Logic for them is the study of *logos*, that is, the study of reason as revealed in articulate speech. Thus, logic as a part of philosophy is meant to examine everything to do with rational discourse.

The Stoics divide logic into rhetoric (*rhetorikē*) and dialectic (*dialektikē*): rhetoric is the art of speaking well in the form of whole, continuous speeches; dialectic, on the other hand, is the art of conducting discussions by means of short questions and answers, but in a much broader sense, it is also defined as the science of what is true, what is false, and what is neither true nor false (D.L. 7.41–42 = LS 31A1–5). Dialectic itself is subdivided into the topics of significations and utterances, that is, it separately studies what is signified by our utterances and the utterances themselves. The study of what is signified covers what gets said by using all sorts of utterances, but mainly by using declarative sentences, and hence propositions, the relations between them, the arguments composed of such propositions, and especially their validity. Often, though, it also covers how we distinguish true from false impressions (*phantasiai*), because on the Stoics’ view it is on the basis of criteria for true impressions that we are able to determine which propositions are true. The study of utterances includes purely linguistic and grammatical phenomena, that is, a physical account of sound appropriately formed by the speech organs, a discussion of the phonemes or letters of the alphabet, an analysis of the parts of speech, an examination of the criteria for good style (D.L. 7.43–44 = LS 31A7–9). Therefore, although “dialectic” is the Stoic term most closely corresponding to our sense of “logic,” the Stoics include under dialectic a good deal that we would call epistemology, philosophy of language, grammar, and linguistics. In what follows we will mainly be concerned with the narrow sense of dialectic that fits, more or less, our modern understanding of logical studies.

Thus defined, Stoic logic aims at a systematic understanding of the rules of rationality, which can assist us to think clearly and correctly, and protect us from being misled

by fallacious arguments in all kinds of rational discourse. In other words, logic is meant to help people discuss and argue correctly, ask and answer questions methodically, explore all the arguments for and against a given thesis, distinguish the true from the false, clarify ambiguous statements, solve paradoxes. In general, the aim of logic is the establishment of a true and stable understanding of the world, an understanding that is supposed to be essential to human beings if they are to live a well-reasoned and ordered life (D.L. 7.46–48 = LS 31B).

Hence, logic turns out to be both a prerequisite for the proper understanding of the physical world and a necessary component of a moral life. Given the Stoics' belief in the rationality of nature, logic becomes inseparable from the other parts of philosophy, and this for the following reason: whereas the end of physics is knowing the world and its order, and that of ethics is living in accordance with the natural order, logic aims at distinguishing the true from the false, and thus makes it possible to find out the truths in the domains of reality which belong to the other parts of philosophy. That is why the Stoics come to understand logic as a particularly important part of philosophy, and that is exactly why they insist that the philosopher must be, more than anything else, a dialectician:

The reason why the Stoics adopt these views in logic is to give the strongest possible confirmation to their claim that the wise man is always a dialectician. For all things are observed through the study conducted in discourses, whether they belong to the domain of physics or equally that of ethics. As to logic, that goes without saying. (D.L. 7.83 = LS 31C)

To show the special role of logic in the interrelation between the three parts of philosophy, the Stoics moreover compare logic to the shell of an egg, to the surrounding wall of a fertile field, to the fortification of a city, or to the bones and sinews of a living being (D.L. 7.40 = LS 26B3; S.E. *M* 7.19 = LS 26D).

Although the Stoics regard logic as a genuine part of philosophy, they certainly do not consider it as a mere auxiliary instrument to ethics and physics. In fact, there seems to have been a considerable dispute over the issue whether logic is a part (*meros*) or merely an instrument (*organon*) of philosophy, a dispute which, although it took place most probably only in late antiquity, helps us to reconstruct the reasoning behind the Stoics' insistence on regarding logic as an integral constituent of philosophy (Alexander, *In Apr.* 1.4–4.29; Ammonius, *In Apr.* 8.15–11.21; Philoponus, *In Apr.* 6.19–9.20). For it becomes clear that, given the subject-matter and aim of Stoic logic, the Stoics have every reason to believe that their logic does not simply provide the other sciences with demonstrative methods. Rather, Stoic logic deals with a particular domain of reality of its own, which is distinct both from the subject-matter of physics and that of ethics, namely it deals with propositions and their interrelations; for, as we will shortly see, the Stoics view propositions as states of affairs which, although they do not exist as bodies do, definitely are part of reality in that they obtain and, if true, are facts. Furthermore, in its concern with truth, Stoic logic goes beyond the bounds of a science aiming only at producing proofs for scientific theorems; among other things, it is meant to enable us to distinguish between the true and the false quite generally.

*The Stoic logicians and their sources*

To better understand the emergence of Stoic logic, it is useful to get an idea of the logical background out of which it historically developed. There are two philosophical schools that could have influenced Stoic philosophers in their logical interests: first, of course, Aristotle and his followers, mainly Theophrastus and Eudemus, and second, the Megarians, like Diodorus Cronus and Philo of Megara.<sup>1</sup>

Indeed, the prevailing view in the nineteenth century was that Stoic logic should be considered as a mere supplement to Aristotle's logical theory; for Stoic logic, so it was alleged, does nothing more than either copy Aristotelian logic or develop it in a vacuous and formal way. It is only since about the middle of the twentieth century, after the important advances in symbolic logic, that it has become obvious how Stoic logic essentially differs from Aristotelian logic. It has even been suggested that the Stoics could not possibly have been influenced by Aristotle, because after the death of Theophrastus Aristotle's esoterical writings, and therefore his logical works, were no longer available and were only recovered in the first century BCE.

I think it is extremely implausible to assume that Aristotle's logical writings were not available to the early Stoic logicians. But, even if the Stoics were familiar with these treatises, there can be no doubt about the originality of the Stoic logical system; just studying Stoic logic and comparing it with what we know about Aristotelian logic provides us with adequate proof that they are two radically different systems. It is true, on the other hand, that Theophrastus and Eudemus published treatises about what they called syllogisms "based on a hypothesis" (*ex hupotheseōs*), which Aristotle (*APr.* I.29, 45b15–20) had promised to write about, but never did, and these syllogisms have a great deal in common with the types of syllogisms the Stoics discuss. However, there is no evidence that the Peripatetic logicians anticipated the outstanding feature of Stoic logic, namely constructing a logical system to prove the validity of a whole class of arguments, though of a different kind than those Aristotle focused on in his syllogistic.

As far as the Megarians are concerned, the historical connections between them and the Stoics are well-documented. Zeno knew both Diodorus and Philo well, and Chrysippus wrote treatises in which he criticized their logical views. However, although Diodorus and Philo were not exclusively concerned with the study of logical puzzles or paradoxes, in connection with which they are usually mentioned, but also put forward original views, for instance about logical modalities and the truth conditions of conditional propositions, they never came close to constructing a logical system as elaborate and sophisticated as that of the Stoics.

But what do we actually know about the Stoic philosophers who were particularly instrumental in the development of Stoic logic? The philosopher who immediately comes to mind, of course, is Chrysippus: "He [Chrysippus] became so renowned in dialectic that it was the general opinion that if the gods had dialectic, it would be no different from that of Chrysippus" (D.L. 7.180 = LS 31Q). Indeed, to confirm Chrysippus' reputation as the principal Stoic logician, one needs only to go through the long list of logical books attributed to him by Diogenes Laertius (D.L. 7.189–198). But Chrysippus

1. On the Megarians, see Declava Caizzi, *MINOR SOCRATICS*, in this volume.

was not alone among the Stoics in his interest in logic. For though it may have been the case that Zeno and Cleanthes, before Chrysippus, were not logicians in the sense that they constructed a formal logical system, they both used valid arguments of a considerable level of logical complexity in order to establish their philosophical theses; and, given the rather standardized patterns of their arguments, one might think that they must have been aware of the logical forms in virtue of which these logical arguments are valid. Also, logical studies in the Stoic school certainly did not die with Chrysippus; there were later Stoic logicians, and they did not simply parrot Chrysippus' doctrines. There is some evidence that Stoic philosophers, like Posidonius, Athenodorus and Epictetus, made further additions to the Chrysippean system, and even that they diverged from Chrysippus' logical theses on lesser issues.

Having said that, however, it is not at all easy to distinguish between Chrysippus' own views and those of his successors, given the state of the surviving evidence; for instance, our ancient sources usually attribute the logical doctrines to the Stoics in general, and not to individual Stoics. Moreover, the fact that most of our sources for Stoic logic are quite hostile and late makes our project of reconstructing the Stoic logical system seem extremely problematic. But it is not impossible. Needless to say, though, there is always plenty of space for different interpretations of the surviving texts, and thus plenty of disagreement among modern scholars on many points of detail.

## The Stoic Logical System

### *Lekta and axiōmata*

The main characteristic of Stoic logic is that the inferences it studies are about relations between items that have the structure of propositions. Whereas Aristotle focused his attention on inferences that involve relations between terms, and thus introduced a logical system similar to what we nowadays call "predicate logic,"<sup>2</sup> Stoic logic marks the beginning of what is now called "propositional logic." To say, though, that Stoic logic is propositional may be somewhat misleading; for, to start with, the Stoics have quite a different understanding of what a proposition is, or to use their own term, of what an *axiōma* (assertible) is: "They say that an *axiōma* is a complete *lekton* which, as far as it itself is concerned, can be asserted" (S.E. *PH* 2.104 = LS 35C2).

So in order to fully grasp the Stoic definition of an *axiōma*, we first need to get some idea about the Stoic notion of a *lekton* (sayable).<sup>3</sup>

### *Lekta* (Sayables)

The term *lekton* is derived from the Greek verb "*legein*," i.e., "to say," and it is, therefore, what has been or gets said or something which can be said. In fact, the Stoics distinguish between what gets said by uttering or using an expression and the

2. On Aristotle's logic, see Detel, *ARISTOTLE'S LOGIC AND THEORY OF SCIENCE*, in this volume.

3. On the sayables in the Stoic theory of meaning, see also Modrak, *PHILOSOPHY OF LANGUAGE*.

expression itself which we utter or use in saying something. For instance, they distinguish between the expression “Cato is walking,” which is used to say that Cato is walking, and what gets said by using this expression, namely that Cato is walking (Sen. *Ep.* 117.13 = LS 33E). So the kind of item which gets said by using the appropriate expression in the appropriate way, the Stoics call a *lekton*.

The Stoics also talk about a *lekton* as the state of affairs signified, i.e., the signification (*sēmainomenon*), distinguishing it from the utterance which is the signifier (*sēmainon*), and from the corporeal entity which the *lekton* is about (*tunchanon*) (S.E. *M* 7.11–12 = LS 33B). Thus *lekta* are items that are placed between mere vocal sounds or written sentences on the one hand and the objects in the world on the other; very roughly speaking, *lekta* are the underlying meanings in everything we say, as well as in everything we think. For *lekta* are defined by the Stoics also as the content of our thoughts: “They say that a *lekton* is what subsists in accordance with a rational impression” (D.L. 7.63 = LS 33F2; S.E. *M* 7.70 = LS 33C).

But not everything that gets thought gets said, and not everything that can be said gets thought. There are indeed many things that never get thought or said, although they are there to be thought or said. In other words, Stoic *lekta* are not mind-dependent items; at the same time, though, they certainly do not exist in the way bodies exist in the world. The Stoics stress that *lekta* are incorporeal, like void, place, and time (S.E. *M* 10.218 = LS 27D), and in order to characterize their mode of being, they introduce the notion of subsistence (*huphistanai*), as opposed to existence (*einai*). Reality, they claim, is not just constituted by corporeal entities, but also by predicates true of bodies and propositions true about bodies. Hence, *lekta* are given in Stoic ontology some status, namely the status, not of bodies, but of incorporeal somethings.

The Stoics divide *lekta* into complete and incomplete (D.L. 7.63 = LS 33F). Incomplete *lekta* include predicates, like for instance what is meant by “writes,” for it is simply a thing to say about something. On the other hand, questions, oaths, invocations, addresses, commands, curses, are all complete *lekta*. But for the Stoic logical system the most important kind of a complete *lekton* is what we call a proposition, for example, that Socrates writes, and this the Stoics call an *axiōma*.

### *Axiōmata* (Assertibles)

So let us return to Sextus’ definition of a Stoic *axiōma*: “They say that an *axiōma* is a complete *lekton* which, as far as it itself is concerned, can be asserted” (S.E. *PH* 2.104 = LS 35C2). And let us also quote the definition that Diogenes attributes to Chrysippus himself: “An *axiōma* is that which is true or false, or a complete state of affairs which, so far as itself is concerned, can be asserted, as Chrysippus says in his *Dialectical definitions*” (D.L. 7.65 = LS 34A). It thus seems that the property of being true or false is what differentiates an *axiōma* from other types of complete *lekta*, but an *axiōma* is mainly defined by the fact that it is the kind of item such that in saying this sort of thing one is asserting something.

Why do we then prefer the translation “assertible” rather than the more common term “proposition?” It is true, of course, that Stoic *axiōmata* and propositions as we conceive them share common characteristics. For instance, they are expressed by complete indicative or declarative sentences, they are either true or false, and they are



incorporeal. But we should also underline the differences between *axiōmata* and propositions. For instance, while a proposition is timelessly true or false, an *axiōma* is asserted at a particular time and has a particular tense; that is to say, an *axiōma* can in principle change its truth-value without ceasing to be the same *axiōma*. For example, the *axiōma* “It is day” is true when it is day and false when it is not (D.L. 7.65 = LS 34E). The *axiōmata* that change their truth-value are called by the Stoics “changing” (*metapiptonta*). For example, the conditional “If Dion is alive, Dion will be alive” is a changing *axiōma*; it is not true at all times, for there will be a time when the antecedent will be true and the consequent false, and thus the conditional will be false (Simpl. *In Phys.* 1299.36–1300.10 = LS 37K). Also, since Stoic *axiōmata* include token reflexive elements, like for instance “this” or “I,” they may cease to exist and presumably also, though this is not clearly stated, begin to exist at definite times. For a Stoic *axiōma* requires the referent of a referring expression as its subject, otherwise it is said to be destroyed; the destruction of an *axiōma* is its ceasing to be expressible. For example, the *axiōma* “This man is dead” is destroyed at Dion’s death, if “this man” refers to Dion (Alexander, *In APr.* 177.25–178.1 = LS 38F).

Being a particular class of *lekta*, *axiōmata* do not exist as bodies do, but they can be said to subsist. In addition, the Stoics make a further distinction: if an *axiōma* is false, it only subsists (*huphistanai*), but if it is true, it is a fact and thus also can be said to be present or there (*huparchein*). In this sense true *axiōmata* correspond to the world’s having certain features, and they are available to be thought and expressed whether anyone is thinking about them or not. On the other hand, since false *axiōmata* are said to subsist, the philosophical question of how false statements and thoughts are possible gets a reasonable answer; false *axiōmata* are the contradictories of facts, and hence have some status.

Finally, *axiōmata* are divided into simple and non-simple *axiōmata*.

### Simple *axiōmata*

According to Sextus, simple *axiōmata* are those which are not composed either of a repeated *axiōma* or of several *axiōmata*; e.g., “It is day” or “Socrates is talking” (S.E. *M* 8.93–98 = LS 34H).

Simple *axiōmata* are again divided into definite (*hōrismena / katagoreutika*), indefinite (*aorista*), and intermediate (*mesa / katēgorika*). Definite *axiōmata* are those that are expressed through demonstrative reference (*deixis*), i.e., through a non-verbal, physical act of indicating something simultaneously with the utterance of a sentence with a demonstrative, e.g., “This one is walking.” Indefinite *axiōmata* are those that are governed by some indefinite constituent, i.e., they are composed by one or more indefinite pronouns and a predicate, e.g., “Someone is walking.” Intermediate *axiōmata* are those that are neither indefinite as to the subject, for they mark off its specific kind, nor definite, for they are not pointing at the subject itself, i.e., they consist of a nominative case and a predicate, e.g., “A man is walking” or “Socrates is walking.”

As for their truth conditions, definite *axiōmata*, such as “This one is walking,” are true whenever the predicate, such as “walking,” belongs to the thing identified by “this one” (S.E. *M* 8.100 = LS 34I). That is to say, of course, that definite *axiōmata* are our means of stating with precision the particular facts in the world. The truth of the

indefinite *axiōmata* is contingent upon those of the corresponding definite ones; for instance, “Someone is walking” comes out true when a definite one of the form “This one is walking” is true (S.E. *M* 8.98 = LS 34H10). It is interesting that the Stoics rejected proper names as subjects of definite *axiōmata*, for on their view proper names signify qualities and could fail to refer; for instance, the “Socraticity” of Socrates marks off a specific individual, but not something necessarily present now, as does “this one.”

The above classification by Sextus does not seem to be an exhaustive division of simple *axiōmata*; it is rather a division of affirmative simple *axiōmata*. Diogenes Laertius proposes instead six classes of simple *axiōmata*, of which the last three classes are similar to the ones discussed by Sextus, while the first three classes are all different kinds of negative *axiōmata* (D.L. 7.69–70 = LS 34K): First, a negative *axiōma* (*apophatikon*) consists of a negative particle and an *axiōma*, e.g., “Not: it is day.” A special case of this is the double negation or super-negation (*huperapophatikon*), which is the negation of a negative *axiōma*, e.g., “Not: not: it is day.” Second, the denial (*arnētikon*), is composed of a denying particle, like “no one,” and a predicate, e.g., “No one is walking.” Third, the privative (*sterētikon*), is composed of a privative particle and a potential *axiōma*, e.g., “This man is unkind,” i.e., “un” and “This man is kind.”

It seems, therefore, that the Stoics think of negative *axiōmata*, as well as of the double negation, as simple *axiōmata*. That is to say, introducing a negative particle does not, by itself, make an *axiōma* non-simple, though negation can also apply to non-simple *axiōmata*. The negation of a simple *axiōma* is itself simple, that of a non-simple *axiōma* non-simple. It is also important to note that the scope of the negative particle is, according to the Stoics, the entire *axiōma*; that means, for instance, that an *axiōma* of the form “It is not day” was regarded as affirmative and not as negative. Hence, the negative particle “not” was not regarded by the Stoic logicians as a connective (*sundesmos*); for such connectives bind together parts of speech, and the negative particle does not do that (D.L. 7.58).

Finally, Stoic negation is truth-functional; that is to say, the negative particle “not,” if added to true *axiōmata*, makes them false, whereas if added to false *axiōmata* makes them true. Moreover, an *axiōma* and its negation form a pair of contradictories (*antikeimena*). Thus, contradictories are *axiōmata* one of which exceeds the other by a negative, provided that the negative is prefixed and controls or governs the whole *axiōma* (S.E. *M* 8.88–90 = LS 34G); e.g., “It is day” and “Not: it is day” are contradictories.

### *Non-simple axiōmata*

Non-simple *axiōmata* are those that are composed either of a repeated *axiōma* or of several *axiōmata* that are combined by one or more connectives (S.E. *M* 8.95). The main types of non-simple *axiōmata* studied by the Stoic logicians are the following (D.L. 7.71–74 = LS 35A; S.E. *M* 8.125–127):

1. A conjunctive *axiōma* (*sumpeplegmenon*) is one which is conjoined by the conjunctive connective “and” or “both . . . and . . .,” e.g., “Both it is day and it is light.” The Stoics gave the obvious truth-conditions for conjunctions, i.e., the truth of the conjunction depends solely on the truth or falsity of the conjuncts, and not on their content. A conjunctive *axiōma* is true if and only if all its conjuncts are

true. However, it seems that this Stoic view was not generally accepted, but was in need of defense. For opponents failed to understand that a conjunctive *axiōma* should be treated as one assertion, and claimed that if some of the conjuncts were true and others false, it naturally should be described as “no more true than false,” though they allowed that it might perhaps be called true if most of the conjuncts were true.

2. A disjunctive *axiōma* (*diezeugmenon*) is one that is disjoined by the disjunctive connective “or” or “either . . . or . . .,” e.g., “Either it is day or it is night.” The Stoics understand the disjunctive relation as exhaustive and exclusive. That is to say, the minimal requirement for the truth of a disjunction was that one and only one disjunct is true.
3. A conditional *axiōma* (*sunēmmenon*) is one linked by the conditional connective “if” (*ei*), e.g., “If it is day, it is light.” A conditional, according to the Stoics, is true when there is “connection” (*sunartēsis*) between the antecedent and the consequent, i.e., when the contradictory of its consequent conflicts with the antecedent. For instance, the conditional “If it is day, it is day” is true, since the contradictory of its consequent “Not: it is day” conflicts with its antecedent “It is day.” A conditional is false when the contradictory of its consequent does not conflict with its antecedent. For instance, the conditional “If it is day, I am talking” is false, since the contradictory of its consequent “Not: I am talking” does not conflict with its antecedent “It is day.”

Hence, Chrysippus assigned to the conditional connective “if” a strong sense, compared to what our sources attribute to Philo of Megara and Diodorus Cronus. For Philo claimed that a conditional is true simply when it does not have a true antecedent and a false consequent, e.g., “If it is day, I am talking.” In fact, this use of the conditional connective “if” is equivalent to what we nowadays call “material implication” and is clearly truth-functional. Diodorus, on the other hand, said that a conditional is true when it neither was nor is able to have a true antecedent and a false consequent. On this view the conditional “If it is day, I am talking” is false, since when it is day but I have fallen silent it will have a true antecedent and a false consequent; but the conditional “If there are no partless elements of things, there are partless elements of things” is true, for it will always have the false antecedent “There are partless elements of things.” On the Stoic view, however, both the conditional “If it is day, I am talking,” and the conditional “If there are no partless elements of things, there are partless elements of things” are false, since in them there is no connection between the antecedent and the consequent (S.E. *PH* 2.110–113 = LS 35B).

The Stoic interpretation of the conditional connective “if” has the disadvantage of rendering at least part of their logic non truth-functional. On the other hand, it is able to adequately express intelligible connections in nature and avoid cases that are counter-intuitive, such as the conditionals “If it is day, I am talking” or “If there are no partless elements of things, there are partless elements of things.” Similarly, the Stoics’ interest in adequately expressing intelligible connections in nature shows in Chrysippus’ decision not to use the conditional when discussing astrological predictions merely based on empirical observation of the correlations between astral and terrestrial events. For example, it may be that it is not the case both that Fabius was born at the rising of

the dog-star and that Fabius will die at sea. Chrysippus would not express this as “If Fabius is born at the rising of the dog-star, he will not die at sea,” precisely because he was not convinced that there was a necessary causal connection between being born at that time of the year and dying on dry land. This is the reason why Chrysippus preferred in such cases the negated conjunction, i.e., “Not: Both Fabius was born at the rising of the dog-star and Fabius will die at sea” (Cic. *Fat.* 12–15 = LS 38E).

Needless to say, non-simple *axiōmata* can be composed of more than two simple *axiōmata*, either because the constituent *axiōmata* are themselves non-simple, or because certain connectives, namely the conjunctive and the disjunctive connective, are two-or-more-place functions. For instance, the Stoics use the conditional “If both it is day and the sun is above the earth, it is light,” or the three-place disjunction “Either wealth is good or it is evil or it is indifferent.”

But we find in our sources more kinds of non-simple *axiōmata*, apart from conjunctions, disjunctions, and conditionals; most of these additional non-simple *axiōmata* may have been introduced after Chrysippus. For instance, a subconditional *axiōma* (*parasunēmnenon*) is one which is joined by the connective “since” (*epei*). A subconditional is true when the consequent follows from the antecedent, and the antecedent holds, e.g., “Since it is day, the sun is above the earth,” when said in daytime; it is false when it either has a false antecedent, or has a consequent which does not follow from the antecedent, e.g., “Since it is night, Dion is walking.” A subdisjunctive *axiōma* (*paradiezeugmenon*) is a non-simple *axiōma* which is indistinguishable in its linguistic form from a disjunctive *axiōma*, but which is true either when its subdisjuncts do not conflict with each other or when the contradictories of its subdisjuncts are not mutually incompatible. For example, the subdisjunctive *axiōma* “Either you are running or you are walking or you are standing” is true, because the contradictories of the disjuncts are not mutually incompatible (Gell. *NA* 16.8.12–14 = LS 35E). A causal *axiōma* (*aitiōdes*) is one which is joined by the connective “because” (*dioti*), e.g., “Because it is day, it is light” (D.L. 7.72). The truth conditions of this kind of non-simple *axiōma* unfortunately have not survived.

### Modalities

Although Stoic logic does not deal with *axiōmata* of the form “It is possible that it is day,” simple and non-simple *axiōmata* are standardly classified as possible, impossible, necessary, and non-necessary. For the Stoic logicians regarded, like truth and falsehood, modalities too as properties of *axiōmata*; so, according to their view, an *axiōma* may in principle change its modal value, since it has it at a time.

Stoic modal logic developed out of the debate over Diodorus Cronus’ famous Master Argument (*kurieuōn logos*), which Epictetus discusses in the following text:

These seem to be the sort of starting-points from which the Master Argument is posed. The following three propositions mutually conflict: “Every past truth is necessary”; “Something impossible does not follow from something possible”; and “There is something possible which neither is nor will be true.” Diodorus saw this conflict and exploited the convincingness of the first two to establish the conclusion that “Nothing which neither is nor will be true is possible.” Now some will retain the pair “There is something possible which neither is nor will be true” and “Something impossible does not follow

from something possible,” but deny that “Every past truth is necessary.” This seems to have been the line taken by Cleanthes and his circle, and was in general endorsed by Antipater. Whereas others will retain the other pair, that “There is something which neither is nor will be true,” and that “Every past truth is necessary,” but hold that something impossible does follow from something possible. To retain all three is impossible because of their mutual conflict. So if someone asks me, “Which of them do you retain?” I shall answer “I don’t know; but my information is that Diodorus retained the first pair I mentioned, the circles of Pathoides (I think) and Cleanthes the second pair, and Chrysippus and his circle the third pair.” (Epict. *Diss.* 2.19.1–5 = LS 38A)

Thus, it seems that the Stoics made various attempts to rebut Diodorus’ view that nothing is possible which neither is nor will be, reacting to the threat of a weakened form of logical determinism entailed by such a claim. In addition, Alexander of Aphrodisias (*In Apr.* 177.25–178.1 = LS 38F) gives us some further information about Chrysippus’ attack on the second premise of the Master Argument. For in order to show that nothing precludes that something impossible follows from something possible, Chrysippus is reported to have used as an example the conditional “If Dion is dead, this one is dead,” and this for the following reasons: First, the antecedent “Dion is dead” is possible, since it will be true at some time. Second, the consequent “This one is dead” is impossible, for any *axiōma* that neither is nor ever can be true is impossible, and “This one is dead” is necessarily either false, namely as long as Dion is alive, or destroyed, namely when Dion is dead. Third, the conditional is true according to all the different truth-conditions for conditionals.

Reacting both towards Diodorus’ definition of the possible as “that which is or will be” and Philo’s definition as “that which is predicated in accordance with the bare fitness of the subject, even if it is prevented from coming about by some necessary external factor” (Alexander, *In Apr.* 183.34–184.10 = LS 38B; Boethius, *Int.* 234.22–26 = LS 38C), Chrysippus proposed his own account of the possible: A possible *axiōma* (*dunaton*) is that which admits of being true, and is not prevented by external factors from being true, e.g., “Dion is alive” (D.L. 7.75 = LS 38D; Boethius, *Int.* 234.27–235.4). Given this definition of the possible, Chrysippus defined the other three standard modal properties of *axiōmata* so as to yield the expected logical relations; for instance, that the necessary is something of which it is not possible that it is not the case, or that the possible and the impossible as well as the necessary and the non-necessary are contradictory to each other.

Hence, an impossible *axiōma* (*adunaton*) is defined as that which does not admit of being true, or admits of being true but is prevented by external factors from being true, e.g., “The earth flies.” A necessary *axiōma* (*anankaion*) is that which is true and does not admit of being false, or admits of being false but is prevented by external factors from being false, e.g., “Virtue is beneficial.” A non-necessary *axiōma* (*ouk anankaion*) is that which is capable of being false, and is not prevented by external factors from being false, e.g., “Dion is walking.”

Two further modalities were also studied by the Stoic logicians; namely, plausibility and probability (D.L. 7.75–76): A plausible *axiōma* (*pithanon*) is that which invites assent to it, e.g., “If someone gave birth to anything, she is its mother.” A probable or reasonable *axiōma* (*eulogon*) is that which has higher chances of being true than false, e.g., “I shall be alive tomorrow.”

### *Arguments*

The Stoics define an argument (*logos*) as a complex or a compound of premises (*lēmματα*) and a conclusion (*epiphora / sumperasma*). The following is a typical Stoic argument (S.E. *PH* 2.135–136 = LS 36B2; D.L. 7.76–77 = LS 36A1–3):

If it is day, it is light.  
But it is day.  
Therefore it is light.

They call the first premise “leading premise” (*hēgemonikon lēmma*: Galen, *Inst. Log.* 7.1), while they call the second premise “co-assumption” (*proslēpsis*). It was the orthodox Stoic view that an argument must have more than one premise, though it seems that Antipater admitted in his logic single-premise arguments (*monolēmματοι*) (S.E. *M* 8.443 = LS 36C7), as for instance (Apul. *Int.* 184.16–23 = LS 36D):

You are seeing.  
Therefore you are alive.

In addition, the Stoics discussed arguments in terms of their modes (*tropoi*), which are the abbreviations of particular arguments; for instance, the mode of the previous argument is the following:

If the first, the second.  
But the first.  
Therefore the second.

The ordinal numbers here stand for *axiōmata*, and have exactly the same role as the letters of the alphabet in Aristotelian logic. Finally, the Stoics also used the so-called “mode-arguments” (*logotropoi*), in which the *axiōmata* are given in full when first occurring, but are then replaced by ordinal numbers, obviously for purposes of simplicity and clarity:

If Plato is alive, Plato is breathing.  
But the first.  
Therefore the second.

Of arguments some are valid or deductive (*perantikoi / sunaktikoi*), others invalid or non-deductive (*aperantoi / asunaktioi*). Invalid arguments occur when the contradictory of the conclusion does not conflict with the conjunction of the premises (D.L. 7.77 = LS 36A4). For instance, the argument:

If it is day, it is light.  
But it is day.  
Therefore Dion is walking.

is invalid, because the contradictory of its conclusion, i.e., “Not: Dion is walking,” does not conflict with the conjunction of its premises, i.e., “Both if it is day it is light and it is day.” In other words, the validity of an argument depends on the truth of the corresponding conditional formed from the conjunction of the premises as antecedent and the conclusion as consequent (S.E. *PH* 2.137 = LS 36B3; S.E. *M* 8.416, 421). To take again the previous argument, it is invalid because the corresponding conditional “If both if it is day it is light and it is day, Dion is walking” is false, at least according to Chrysippus’ truth-conditions for conditional *axiōmata*.

Further, some arguments are true, others false (D.L. 7.79 = LS 36A8–9; S.E. *PH* 2.138–139 = LS 36B4–6). True are those arguments which deduce correctly from true premises, e.g.,

If virtue benefits, vice harms.  
But virtue benefits.  
Therefore vice harms.

False arguments are those which either have some false premise, or are invalid, e.g.,

If it is day, it is night.  
But it is day.  
Therefore Dion is alive.

#### Syllogistic arguments

Of valid arguments, some are just called “valid,” others “syllogistic” (*sullogistikoi*). The Stoics define syllogistic arguments as those which either are what they call “indemonstrable” (*anapodeikttoi*), or can be reduced to the indemonstrables (D.L. 7.78 = LS 36A5).

#### *Indemonstrables or simple syllogisms*

Indemonstrable arguments are those whose validity is not in need of demonstration, given that it is obvious in itself (D.L. 7.79–81 = LS 36A11–16; S.E. *M* 8.223–227; *PH* 2.157–158). The lists of indemonstrable arguments which are to be found in our ancient sources vary, but there is no doubt that Chrysippus himself distinguished five different types of such arguments, each type being characterized by a particular basic form in virtue of which the arguments are understood to be indemonstrable. As to other surviving lists of types of indemonstrables, they reflect the criticisms and alterations that the standard list seems to have undergone at the hand of later Stoics in the centuries following its introduction; they differ from Chrysippus’ list in adding extra types of argument, as well as in objecting to the usefulness or application of others (Cic. *Top.* 57; Martianus Capella 4.419–420; Boethius, *Cic. Top.* 358; Cassiodorus, *Inst.* 119.3–4).

The basic logical forms of the five standard indemonstrables are described and illustrated by examples in our texts as follows:

1. A first indemonstrable argument is constructed out of a conditional and its antecedent as premises, and the consequent as conclusion, e.g.,

If it is day, it is light.  
 But it is day.  
 Therefore it is light.

2. A second indemonstrable argument is constructed out of a conditional and the contradictory of its consequent as premises, and the contradictory of its antecedent as conclusion, e.g.,

If it is day, it is light.  
 But not: it is light.  
 Therefore not: it is day.

3. A third indemonstrable argument is constructed out of a negated conjunction and one of its conjuncts as premises, and the contradictory of the other conjunct as conclusion, e.g.,

Not: both Plato is dead and Plato is alive.  
 But Plato is dead.  
 Therefore not: Plato is alive.

4. A fourth indemonstrable argument is constructed out of a disjunction and one of its disjuncts as premises, and the contradictory of the other disjunct as conclusion, e.g.,

Either it is day or it is night.  
 It is day.  
 Therefore not: it is night.

5. A fifth indemonstrable argument is constructed out of a disjunction and the contradictory of one of its disjuncts as premises, and the other disjunct as conclusion, e.g.,

Either it is day or it is night.  
 Not: it is day.  
 Therefore it is night.

Of course, the types of indemonstrable arguments include many more arguments than the simple examples above suggest. For instance, if in each type of indemonstrable the simple *axiōmata* that we use to construct the premises are not affirmative, as they are in the above simple examples, but either both negative or negative and affirmative respectively, we get different combinations of premises, and hence different indemonstrable arguments of the same type. That is to say, in the case of the first indemonstrable, if, instead of having as first premise the conditional “If it is day, it is



light,” we use the conditionals “If not: it is day, not: it is light” or “If not: it is day, it is light” or “If it is day, not: it is light,” together with the appropriate affirmative or negative *axiōmata* as second premises, we still get different arguments of the same first type of indemonstrables. In addition, it becomes clear that many more arguments are included in each type of the five indemonstrables, if we consider the cases in which the *axiōmata* of the premises are themselves non-simple, as, for instance, in the case of the Chrysippean argument discussed in the following text:

According to Chrysippus (that arch-enemy of irrational animals!) the dog even shares in their legendary “dialectic.” At any rate, this man says that the dog applies himself to a multiple fifth indemonstrable when he comes to a triple fork in the path, and, after sniffing the two paths which his quarry did not take, sets off at once down the third without even sniffing it. For, the ancient philosopher says, the dog is in effect reasoning: “Either my quarry went this way, or this way, or this way. But neither this way, nor this way. Therefore this way.” (S.E. *PH* 1.69 = LS 36E)

But why did Chrysippus decide to suggest this particular list of the five types of indemonstrable arguments? It is certainly not the case that Chrysippus was trying to introduce the smallest possible number of different types of indemonstrable arguments. For one could easily dispense with the second indemonstrable on the basis of the first indemonstrable and the logical principle which the Stoics call “the first *thema*,” a principle of contraposition of the conclusion with a premise which we will shortly discuss; similarly, the fifth indemonstrable seems to be redundant, since we already have the fourth indemonstrable, if we avail ourselves again of the first *thema*. Rather, it has reasonably been suggested that Chrysippus included in his list of the five indemonstrables all types of argument which just rely on the argumentative force of the different basic types of connectives known to him. In the case of the third indemonstrable, for instance, to use a negated conjunction just is to say that if one of the conjuncts holds the other does not; or again, the fourth and fifth indemonstrables just rely on what it means to use the disjunctive connective, namely to say that if one of the disjuncts holds the contradictory of the other holds too, and if the contradictory of one disjunct holds the other disjunct holds too.

#### *Non-simple syllogisms*

But not only are the indemonstrable arguments, according to the Stoics, syllogistic; our sources discuss Stoic examples of syllogistic arguments which are not themselves indemonstrable. For instance, we find the following example of a syllogistic argument which, though it also has two premises and a conditional as its first premise, is more complex than the first or the second indemonstrables (S.E. *M* 8.230 = LS 36G3):

If it is day, if it is day it is light.  
But it is day.  
Therefore it is light.

We also find a Stoic example of a non-simple syllogism with three premises (S.E. *M* 8.234 = LS 36G6):

If things evident appear alike to all those in like condition and signs are things evident,  
 signs appear alike to all those in like condition.  
 But signs do not appear alike to all those in like condition.  
 And things evident do appear alike to all those in like condition.  
 Therefore signs are not things evident.

In fact, given the complexity of this syllogistic argument, Sextus also gives us its mode (S.E. *M* 8.235 = LS 36G7):

If both the first and the second, the third.  
 But not the third.  
 But also the first.  
 Therefore not the second.

Another Stoic example of a non-simple syllogism with three premises is the following (S.E. *M* 8.466; *PH* 2.186; cf. *M* 8.281):

If proof exists, proof exists.  
 But if proof does not exist, proof exists.  
 But also either proof exists or does not exist.  
 Therefore proof exists.

And we are again given its mode (S.E. *M* 8.292):

If the first, then the first.  
 But if not the first, then the first.  
 But also either the first or not the first.  
 Therefore the first.

Syllogistic are also, according to the Stoics, the so-called “indifferently concluding arguments” (*adiaphorōs perainontes logoi*), of which our ancient sources provide us the following example (Alexander, *In Top.* 10.10–12):

Either it is day, or it is light.  
 But it is day.  
 Therefore it is day.

Besides these non-simple syllogisms, there are also the so-called “duplicated arguments” (*diaphoroumenoi logoi*); their first premise is a non-simple *axiōma*, which is constructed out of the same simple *axiōma* used twice or more times (Alexander, *In Top.* 10.7–10; D.L. 7.68–69; S.E. *PH* 2.112), e.g.,

If it is day, it is day.  
 But it is day.  
 Therefore it is day.

Finally, arguments of the following kind are also regarded by the Stoics as syllogistic (Origen, *Cels.* 7.15 = LS 36F):

If you know that you are dead, you are dead.  
 But if you know that you are dead, not: you are dead.  
 Therefore not: you know that you are dead.

All these non-simple syllogisms, the Stoics seem to have held, are formed by combination of simple syllogisms or indemonstrables, and thus they can be reduced to them by purely logical means. Indeed, to demonstrate the syllogistic validity of any argument whatsoever, the Stoic logicians considered it necessary to reduce it to one or more of the indemonstrable arguments. This procedure of reducing non-simple syllogisms to indemonstrable arguments was called by the Stoics “*analysis*.”

### *Analysis*

To go by its very name, analysis is a method of reducing something to something more basic or prior, i.e., in this case a method of reducing the non-simple syllogisms to the indemonstrables, which are regarded as the first principles of the Stoic logical system. To carry out this procedure, the Stoic logicians had, according to our ancient sources, at least four logical rules which were called “*themata*” (D.L. 7.78 = LS 36A5; Galen, *Plac.* 2.3.18–19 = LS 36H), and in Latin “*constitutiones*” or “*exposita*” (Apul. *Int.* 191.5–10 = LS 36I). We only know the first and the third Stoic *thema*, and it is on the basis of extremely meager evidence that modern scholars have suggested their different reconstructions of the other two.

The first *thema* is the following:

If from two propositions a third is deduced, then from either one of them together with the contradictory of the conclusion the contradictory of the other is deduced. (Apul. *Int.* 191.6–10 = LS 36I)

It can be formalized in the following way:

$$\frac{P_1, P_2 \vdash C}{P_1, \text{ctrd } C \vdash \text{ctrd } P_2}$$

The third *thema* is the following:

When from two propositions a third is deduced, and extra propositions are found from which one of those two follows syllogistically, the same conclusion will be deduced from the other of the two plus the extra propositions from which that one follows syllogistically. (Alexander, *In Apr.* 278.12–14 = LS 36J; cf. *Simpl. In Cael.* 237.2–4)

It can be formalized in the following way:

$$\frac{P_1, P_2 \vdash C; E_1, \dots, E_n \vdash P_2}{P_1, E_1, \dots, E_n \vdash C}$$

As to the second and fourth *themata*, we try to reconstruct them mainly on the basis of a logical principle, the so-called “dialectical theorem” (*dialektikon theōrēma*) or “synthetic theorem” (*sunthetikon theōrēma*), which is most probably Peripatetic and which is supposed to do, according to our ancient sources (Alexander, *In Apr.* 284.10–17), the same job as the second, third, and fourth *themata* together:

When we have the premises from which some conclusion is deducible, we potentially have that conclusion too in these premises, even if it is not expressly stated. (S.E. *M* 8.231 = LS 36G4; cf. Alexander, *In Apr.* 274.21–24; 278.8–11; 283.15–17)

Indeed, the only two examples of Stoic analysis that have survived, and are both reported by Sextus Empiricus, make use of this dialectical theorem in order to reduce certain non-simple syllogisms to indemonstrable arguments, and thus prove their syllogistic validity. In particular, the first example of analysis (S.E. *M* 8.232–233 = LS 36G5) deals with the first non-simple syllogism which we mentioned earlier, namely:

If it is day, if it is day it is light.  
But it is day.  
Therefore it is light.

The second example of analysis (S.E. *M* 8.235–236 = LS 36G7) deals with the second non-simple syllogism that we have previously mentioned, namely:

If things evident appear alike to all those in like condition and signs are things evident,  
signs appear alike to all those in like condition.  
But signs do not appear alike to all those in like condition.  
And things evident do appear alike to all those in like condition.  
Therefore signs are not things evident.

Let us then discuss in detail the second of these examples, so that we get a clearer idea of how Stoic analysis actually functions, and how we are supposed to apply the Stoic *themata*. To this purpose, it would be easier to use, as Sextus himself does, the mode of the non-simple syllogism, namely:

If both the first and the second, the third.  
But not the third.  
But also the first.  
Therefore not the second.

Sextus suggests that this argument can be reduced to two indemonstrables of different types, namely to a second and a third indemonstrable argument, by going through the following two steps:

1. By combining the first premise, which is a conditional, with the second premise, which is the contradictory of the conditional’s consequent, we get a second indemonstrable which has as its conclusion the contradictory of the conditional’s antecedent:

If both the first and the second, the third.  
 But not the third.  
 Therefore not: both the first and the second.

2. By combining the conclusion of this indemonstrable, which is a negated conjunction, with the third remaining premise, which affirms one of the two conjuncts, we get a third indemonstrable which has as its conclusion the affirmation of the other conjunct:

Not: both the first and the second.  
 But the first.  
 Therefore not: the second.

Hence, the dialectical theorem in this case validates the use of the conclusion of the second indemonstrable, that is to say the use of the negated conjunction, in the construction of the third indemonstrable; for, according to this logical rule, the negated conjunction which is deduced from some of the premises of the argument is implicitly contained in the argument, though it is not expressly stated. And it is obvious that we may similarly use the third *thema*; for a single application of the third *thema* on the second and third indemonstrables, which we have constructed, could help us deduce the non-simple syllogism whose validity we try to prove.

To summarize, Stoic analysis starts with a non-simple syllogism and continues with a series of arguments which are either indemonstrables or arguments directly derived from the indemonstrables by appropriate application of one of the Stoic *themata*. Indeed, there are several ancient texts that suggest that the Stoic logicians, and in particular Chrysippus, believed that their standard list of five indemonstrables is complete in the sense of containing all that is required for reasoning. It is said, for instance, that every argument is constructed out of these indemonstrables (D.L. 7.79 = LS 36A11), and that all other arguments are thought to be validated by reference to them (S.E. *PH* 2.156–157; 166–167, 194). Therefore, we may certainly infer that some claim of completeness was made by the Stoic school, but it is not at all clear what precisely the Stoics' definition of completeness was, if they ever offered one.

#### *Valid arguments, in the narrow sense*

After all, the Stoics themselves admit that we cannot apply the method of analysis to all valid arguments, that is to say we cannot reduce all valid arguments to the five indemonstrables by using the four Stoic *themata*. For, as we have already said, there are arguments in Stoic logic which are just valid, but not syllogistic (D.L. 7.78 = LS 36A6). It seems that, according to the Stoics, the validity of such arguments is guaranteed not by their own analysis, but by their being equivalent to syllogistic arguments. To explain what I mean, let us briefly discuss the two groups of arguments of this kind for which we have some evidence: the so-called “subsyllogistic arguments” (*huposullogistikoi logoi*), and the “unmethodically conclusive arguments” (*amethodōs perainontes logoi*).

Subsyllogistic arguments differ from syllogisms only in that one or more of their constituent *axiōmata*, although being equivalent to those in a syllogism, diverge from them in their linguistic form (Gal. *Inst. Log.* 19.6). The example given by Diogenes Laertius to illustrate the class of valid arguments in the narrow sense most probably is a subsyllogistic argument; for if it were not for the first premise which slightly diverges from the linguistic form of a negated conjunction, the argument would have been a third indemonstrable (D.L. 7.78 = LS 36A6):

“It is day and it is night” is false.  
But it is day.  
Therefore not: it is night.

Alexander of Aphrodisias also seems to discuss an example of a subsyllogistic argument in the following text:

For while “If A then B” means the same as “B follows from A,” they [i.e. the Stoics] say that there is a syllogistic argument if we take the expression “If A then B. But A. Therefore B,” but that the argument “B follows from A. But A. Therefore B” is not syllogistic but concluding. (Alexander, *In APr.* 373.31–35)

In general, what emerges from the Stoics’ treatment of subsyllogistic arguments is that the Stoic logicians tried to eliminate unnecessary ambiguities by standardizing language, so that the form of a sentence would unambiguously determine the type of *axiōma* expressed by it. For one and the same sentence may express *axiōmata* that belong to different classes, and equally two different sentences may express the same *axiōma*. But if there is some agreement to fix language use in a certain way, it becomes possible to easily discern the logically relevant properties of *axiōmata* and their compounds by simply examining the linguistic expressions used.

The case, however, of the unmethodically concluding arguments is more perplexing; for it is difficult to be certain about the actual kinds of argument which belong to this group as well as about the reasons on the basis of which the Stoics consider them as valid. For instance, it seems that the following argument is a Stoic example of an unmethodically concluding argument (Alexander, *In APr.* 22.17–23; 345.28–29; *In Top.* 14.27–15.3; Gal. *Inst. Log.* 17.2):

Dio says that it is day.  
But Dio speaks truly.  
Therefore it is day.

On the other hand, our evidence is unclear as to whether the following unmethodically concluding argument is of Stoic origin (Alexander, *In APr.* 21.31–22.7; 344.14–15; *In Top.* 14.21–27):

A is equal to B.  
C is equal to B.  
Therefore A is equal to C.

And similarly, in the following case (Alexander, *In Apr.* 344.31–34):

A has the same parents as B.  
B has the same parents as C.  
Therefore, A has the same parents as C.

Moreover, it is uncertain which the corresponding syllogisms are on the basis of which these arguments are, according to the Stoics, valid. For the first example Alexander of Aphrodisias (*In Apr.* 345.30–346.4) suggests that we add an appropriate universal premise, while at the same time combining the existing two premises into one, so that one may construct the following syllogism:

Everything that someone says when speaking truly is the case.  
Dio, speaking truly, says that it is day.  
Therefore it is day, as Dio says.

But it is difficult to see why this would be a satisfactory solution for the Stoic logicians.

### *Invalid arguments*

According to the Stoics, invalid arguments arise in four ways: by disconnection, by redundancy, by being propounded in an invalid schema, and by deficiency (S.E. *M* 8.429–434 = LS 36C1–5):

1. Arguments are invalid by disconnection (*kata diartēsin*) when premises have no connection with one another or with the conclusion, e.g.,

If it is day, it is light.  
But wheat is being sold in the market.  
Therefore it is light.

2. Arguments are invalid by redundancy (*kata parolkēn*) when they contain premises which are superfluous for drawing the conclusion, e.g.,

If it is day, it is light.  
But it is day.  
But also virtue benefits.  
Therefore it is light.

3. Arguments are invalid by being propounded in an invalid schema (*en mochthērōi schēmati*), e.g.,

If it is day, it is light.  
But not: it is day.  
Therefore not: it is light.

4. Arguments are invalid by deficiency (*kata elleipsin*) when they contain premises that are incomplete, e.g.,

Either wealth is bad, or wealth is good.  
 But not: wealth is bad.  
 Therefore wealth is good.

For the first premise should be “Either wealth is bad, or wealth is good, or wealth is indifferent”; and moreover, a premise is missing.

### *Paradoxes*

Finally, the Stoics discussed some arguments which they called “sophisms,” and among which we also find what we nowadays consider as logical paradoxes:

A sophism (*sophisma*), they say, is a plausible argument deceitfully framed to make us accept the false or false-seeming or non-evident or otherwise unacceptable conclusion.  
 (S.E. *PH* 2.229 = LS 37A2)

There is abundant evidence of the Stoics’ interest in solving logical paradoxes; just going through the list of Chrysippus’ logical works shows that he in particular took them very seriously (D.L. 7.192–198 = LS37B). After all, the surprising conclusions of these seemingly rather simple arguments were a challenge to the Stoics’ conception of basic logical notions, such as truth and falsehood, so that it soon became clear that solving the paradoxes would require careful reappraisal of some parts of their logical system. More specifically, the Stoics were intrigued by the paradoxes which had already puzzled the Megarians, for instance the Liar and the *Sorites*.

#### The Liar

Various versions of the Liar paradox were known in antiquity, but there is no single text that gives us with certainty the precise formulation of the argument. For instance, the following passage from Cicero’s *Academica*, which most probably is our oldest testimony on the Liar, has a devastating lacuna at a crucial point: “If you say that you are lying, and you say so truly, are you lying \*\*\* telling the truth?” (Cic. *Acad.* II.95 = LS 37H5; cf. Gell. *NA* 18.2.9–10).

Different suggestions have been made in order to emend the text in a satisfactory way; one such plausible suggestion which fills up the lacuna and presents us with a real paradox reads as follows: “If you say that you are lying, and you say so truly, you are lying, and if you are lying, you are telling the truth.” Presumably it was Eubulides who invented this paradox in the fourth century, and a version of it seems to have been known to Aristotle (D.L. 2.108). But there is no doubt that it was Chrysippus who more than anyone else in ancient times tried to solve it. For it is clear from our ancient sources that Academic Skeptics in his day used this paradox to challenge the Stoic view that all *axiōmata* are either true or false, and thus to question the Stoic logicians’ faith in the Principle of Bivalence (Cic. *Acad.* II.95 = LS 37H5; Plut. *Comm. not.* 1059D–E = LS 37I).



But did Chrysippus find a solution to the Liar paradox? Some modern scholars have claimed that Chrysippus is a forerunner of the medieval *cassantes*, i.e., those who believed that the Liar's statement "I am lying" is not an *axiōma*, because it is meaningless. Against this, it has been convincingly argued that Chrysippus must have thought that such a statement is an *axiōma*, since there are cases in which it has a clear-cut meaning and a definite truth-value, given the conditions under which it is uttered. The peculiarity of the case of the Liar, at least as it is presented by Cicero, seems to be, not only that we are not able to find out what the truth of the statement is, but that in this case there is no truth of the matter. So perhaps Chrysippus' view was that in cases like this the statement is neither true nor false. However, if this is correct, the solution would put the very notion of an *axiōma* under great pressure and would force a reconsideration of its definition.

### The Sorites

The name of the *Sorites* comes from the Greek noun "*sōros*," which means "heap" or "pile." As it becomes clear from the following text, this paradox exploits the vagueness of certain predicates, like for instance "heap":

Wherefore I say: tell me, do you think that a single grain of wheat is a heap? Thereupon you say No. Then I say: what do you say about 2 grains? For it is my purpose to ask you questions in succession, and if you do not admit that 2 grains are a heap then I shall ask you about 3 grains. Then I shall proceed to interrogate you further with respect to 4 grains, then 5 and 6 and 7 and 8; and I think you will say that none of these makes a heap. Also 9 and 10 and 11 are not a heap . . . If you do not say with respect to any numbers, as in the case of the 100 grains of wheat for example, that it now constitutes a heap, but afterwards when a grain is added to it, you say that a heap has now been formed, consequently this quantity of corn becomes a heap by the addition of the single grain of wheat, and if the grain is taken away the heap is eliminated, And I know of nothing worse and more absurd than that the being and not-being of a heap is determined by a grain of corn. (Gal. *Med. exp.* 17.1 = LS 37E3)

According to Cicero, Chrysippus claimed that this paradox does not pose any real difficulty, because the wise man knows at which moment he should stop replying to questions of the form "Are so-and-so many grains a heap?":

"That doesn't harm me," he says, "for like a skilled driver I shall restrain my horses before I reach the edge, all the more so if what they're heading towards is a precipice. In like manner I restrain myself in advance and stop replying to sophistical questions." (Cic., *Acad.* II.94 = LS 37H3)

In addition, the Stoic logicians concerned themselves with sophisms of the following kinds, which at least are far less problematic:

### The Veiled Man

Chrysippus: Next you're going to hear the quite fascinating Veiled Argument. Tell me, do you know your own father? Customer: Yes. Chrysippus: Well, if I place someone veiled in front of you and ask "Do you know this person?" what will you say? Customer: Obviously

that I don't know him. Chrysippus: But in fact this person is your very own father. So if you don't know this person, you clearly don't know your own father. (Lucian, *Vit. auct.* 22 = LS 37L)

### The Horned Man

If you have not lost something, you have it still. But you have not lost horns. Therefore you still have horns. (D.L. 7.187)

Unfortunately, there is no evidence as to the way in which the Stoics tried to solve these sophisms.

## Conclusion

As I indicated at the beginning of the chapter, it was only towards the middle of the twentieth century that Stoic logic began to be studied on its own merits and not as an appendix to Aristotle's syllogistic. To a great extent it was the revival of interest in the logical contributions of the Stoics that convinced scholars to investigate more carefully the other parts of Stoic philosophy, namely ethics and physics. The literature on Stoic logic that has since been published has managed to reconstruct a logical calculus, which still surprises us with its sophistication and its similarities to modern systems of logic. At the same time, though, it also has become clear that we should not fail to take seriously into account what differentiates Stoic logic from its modern counterparts. For only in this way can we get a better understanding of how the history of logic has evolved in close connection to the other parts of philosophy, and more importantly, only in this way do we have a chance to appreciate the peculiar features and insights of ancient logic.<sup>4</sup>

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